

Understanding Financial Risk Management

Third Edition

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Understanding Financial Risk Management

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INVESTOR IN PEOPLE

“Education breeds confidence. Confidence breeds hope. Hope breeds peace.”

Confucius

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Preface to the Second Edition

The second edition of *Understanding Financial Risk Management* aims to improve the first edition by introducing a more structured approach to the sources of risk in the organization, and the methods used to manage it.

From identification to assessment and management, all types of financial risks a company faces daily are analyzed, together with the tools and techniques that can be used to limit their impact and manage their connected risk events.

Built on the solid pedagogical approach used in the first edition, the second edition improves it by extending the narrative to modern and innovative topics like enterprise risk.

The result is a 22-chapter textbook that takes the student into a full-immersion experience. After an introductory part where distributional issues, statistical tools and other foundation topics are analyzed, the chapters start digging deep into all types of financial risk that are normally presented to the organization on a daily basis.

An improved coverage of major risks, together with ample narrative on how to use financial derivatives to hedge risk, offer a complete view on past, current and future trends in financial risk management.

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Addendum to the Preface

The third edition of *Understanding Financial Risk Management* features a separated chapter for high-frequency data, as well as two new chapters, on commodity risk and digital finance and risk, respectively.

A general refinement has been done throughout the text in terms of editing and corrections, in order to improve the learning experience of the students and enhance the understanding of the various topics.

Overall, the book aims to keep a solid foundation of theory and applications, while looking forward to include modern topics and introduce the students to the digital era and other innovations.

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Chapter 1

Risk: An Overview

Financial markets are typically characterized by transparent pricing and specific trading regulations. The costs, fees and market forces determine the price of traded securities and the risk embedded in any single trade.

Risk can be defined as the possibility of a negative outcome as a consequence of specific choices. In all fields of life, including business and financial activities, actions that lead to a loss can be defined as risk. Various types of daily risk affect a business, but in regard to money, they can be mainly classified into two types: business risk and financial risk.

Financial risk in particular involves the financial structure of the corporation. It generally arises from the variability of prices and returns on financial markets. Movements can involve any kind of security, such as stocks, currencies, derivatives and interest rates.

Financial risk managers have the duty to perform identification, measurement and hedging of risk, if necessary. Financial instruments can be used for this purpose, but it is not always necessary to do so since risk often entails the opportunity of a good expected return in exchange.

It is not possible to prevent all possible risks, given that some of them are unpredictable and not identifiable sufficiently in advance. However, for many types of risk, the analytical, computational and numerical tools available in the literature can offer a way of reducing uncertainty.

After studying this chapter, you will be able to answer the following questions, among others:

- (1) What is financial risk and how does it differ from other types of risk?
- (2) What are the various types of financial risk and how are they linked to each other?
- (3) What are the differences among risk in banks, corporations and insurance?
- (4) How can we define the process of risk management and what steps does it involve?
- (5) What are the most common types of strategies and instruments involved in risk management?

The first section of the chapter is an introduction to the definition of risk and how it arises in different types of business. The second section is about the various types of financial risk and their interconnection. The final part introduces the process of risk management, such as identification, measurement and mitigation and/or transfer.

1.1. Introduction

Learning outcomes:

- (1) Distinguish between uncertainty and risk.
- (2) Learn about generic concepts in utility theory.
- (3) Acquire generic knowledge about different types of risk.

1.1.1. Randomness and Uncertainty

Financial risk management has its roots in the history of commercial and financial trades, but it is still a modern quantitative discipline. Its development began during the 1970s, on the premises of the first Basel Accord, between the G10 countries, which covered the regulation of banking risk.

Modern financial markets are constantly subject to speculative attacks and external shocks, given the uncertainty of the actual worldwide economic environment. Financial risk management is therefore in a state of confusion and has gained focus in recent years on the minimization of capital charges and corporate risk.

Managing risk is about making decisions under uncertainty, with decisions involving a large share of investors, and determining the outcome of investing strategies for even larger investors, such as banks (Bessis, 2010).

It is now clear that old-fashioned financial risk management has contributed to generating the most severe financial crisis the world has ever experienced due to the common use of static risk measurement methodologies and unrestrictive regulatory provisions. As a consequence of the lack of regulation and modeling efficiency, the policies of banks and financial institutions fed the bubble until it burst, with the effects of the explosion spreading all over the world (Nelson & Katzenstein, 2012).

The subprime mortgage affair in the USA turned quite fast into a global crisis involving all sectors of the financial and real economy. At some point, it was clear that something was wrong in how risk assessment of new assets had been managed until then.

One of the first definitions of the difference between uncertainty and risk was given by Knight (1921). He argued that uncertainty differs from risk for the important reason that risk can be measured precisely.

This difference also plays a crucial role in financial markets, given that if risk were the only relevant feature of randomness, it could have been possible for financial institutions properly equipped to price and market insurance contracts based on risky phenomena only. The role of uncertainty is to create frictions that cannot be easily accommodated (Eeckhoudt et al., 2005).

Uncertainty then refers to the situation where an event has an unknown probability and individuals tend to choose gambles with precise expected outcomes compared to gambles with unknown odds.

Both uncertainty and risk define a random environment and affect individuals and their choices. The behavior of individual agents is often counterintuitive if confronted with the classic expected utility model, and research has focused on that aspect.

Based on the fact that, if uncertainty is so influential on individual behavior, an equilibrium outcome should exist, Knight claims that, as opposed to risk, uncertainty cannot be insured.

The presence of uncertainty causes departure from standard utility theory and sets an environment where insurance markets may breakdown, while the randomness of probabilities does not allow for precise pricing of risky claims.

Randomness drives risk and uncertainty and the two concepts are strictly linked to each other but with some differences. Risk entails the existence of a measurable probability associated with the event.

Probabilities are calculated directly or by induction, depending on the availability of observable variables. Calculation by induction is made via analytical models or by analysis of past information.

In other words, risk is a quantifiable variable, while uncertainty is not. This is the main distinction between risk and uncertainty. The purpose of financial analysis is to assess risk based on available information.

When a choice must be made, both uncertainty and risk apply. In the financial literature, most of the work has been done on risk, given its calculability. It is not appropriate to fully ignore uncertainty.

Example 1.1: Consider two portfolios traded on a market. The first portfolio, *A*, is a risk-free investment on government bonds, while the second portfolio, *B*, includes risky complex securities written on a market index. If the expected return on portfolio *A* is 2.5% and the expected return on portfolio *B* is 9.5%, investors are demanding an extra 7% to move their money from a risk-free investment to a risky investment.

The concept of risk spans different disciplines from insurance and engineering to classical economic theories such as portfolio theory. Each discipline defines risk in a different way. Some of these definitions are as follows:

- (a) *Probability and consequences:* This definition of risk focuses on the likelihood of an event happening and the consequences of the event. Both aspects are involved in ranking the risk level of an event. Consider, for example, a tsunami: it has a very small probability of happening, but when happening, it will carry severe consequences. Therefore, it would be ranked as a high-risk event.
- (b) *Risk or threat:* There is a difference between a threat and a risk. Threat is considered to be a very low probability event with extremely large negative outcomes and no possibility for analysts to assess the probability of that event happening. Risk, on the other hand, is similar, but it involves events happening at a higher probability, where it is possible to assess both probability and outcome.
- (c) *Positive and negative outcomes:* Definitions of risk may involve both positive and negative variabilities around the mean or just focus on the downside scenarios. A fair definition of risk in this sense is the product of the probability

of an event occurring and the assessed value of the outcome. In the following chapters, it will be clear how this is the main definition of risk underlying the financial analysis.

To be more specific, in finance, risk is defined as the (positive or negative) variability of financial returns on some type of investment around some expected (mean) return. It is then fundamental to understand how to define risk broadly to include both the positive and negative sides of the variability.

The general rule of thumb is that there is a direct link between risk and reward. High levels of positive outcome can be obtained only by being exposed to considerably large risk. The principle is that there is no free lunch in life, especially in finance. In fact, the link between risk and return is a foundation of classical and modern finance theory.

In the following chapters, this concept will become very clear. Just for now, it is preliminarily important to consider, as an example, the difference between investing in financial markets. Stocks are much riskier than bonds and give a higher expected return.

The concept can be extended to all aspects of life. Therefore, for any business, the key to success is the reliability of the decision on what types of risk is worth taking and how much of it is.

Protecting the business against all the risks, in some sense, results in limiting the profit opportunities. On the other hand, being exposed to the wrong type of risk can lead to even worse scenarios.

That is why a crucial part of good business management is about making the right choices on how to face different types of risk. Good risk management is a crucial part of good corporate management.

A typical example of risk and uncertainty in finance is the classical portfolio problem. Consider an agent endowed with a wealth of amount w to invest. The market is very simple, being composed of a risk-free asset paying a fixed rate r and a risky asset with random return x . The random return is distributed according to a cumulative density function $F(x)$. The utility function of investor u is assumed to be concave (this is very important for the existence of a risk measure, as it will become clearer in the next sections).

The total wealth is invested by the individual by dividing it between an amount m of the risky asset and an amount $w - m$ in the risk-free asset. Therefore, the resulting portfolio value is given by the combination of the stochastic return on the risky asset and the deterministic return on the risk-free asset and can be written as

$$p = mx + (w - m)r$$

The problem of asset allocation among the assets in the market entails an optimization program that maximizes the expected utility from the investment strategy as described.

The investor maximizes the portfolio value given the utility function, and the program is defined as

$$\max \int u[mx + (w - m)r]dF(x)$$

The first-order condition for the program is

$$\int u'[m(x-r) + wr](x-r)dF(x) = 0$$

If the investor is risk neutral, there is no need for compensation for the risk taken, and the resulting utility function is linear and directly proportional to the value of the risky asset, in the form

$$u(x) = \lambda x$$

where λ is some constant.

Thus, the marginal return on the investment is given by the sum of returns on both the risk free and the risky part of the investment.

$$r_M = \lambda r(w - m) + \lambda m E(x)$$

where $E(x)$ is the expected return on the risky asset, which is equivalent to

$$r_M = \lambda wr + \lambda m[E(x) - r] \quad (1.1)$$

Equation (1.1) shows that returns are always positive if $E(x) > r$ and always negative otherwise. The logical consequence is that a risk-neutral investor will always invest all the wealth in the asset with the highest expected return.

Given this result, the concavity of the utility function also implies that the marginal return given by investing slightly more in the risky asset is always positive. This is a very important result, implying that also a risk-averse investor will choose to not put all the wealth on just the risk-free asset.

All types of investors will always choose to put at least a small bit of their wealth on risky assets. In terms of insurance, this means that a risk-averse agent will never choose to buy full insurance unless insurance prices are equal to or below the fair actuarial level, where the fair actuarial price is the price corresponding to a zero net present value.

1.1.2. Rationality and Risk Aversion

The theory of rational expectations is a foundation of modern economics and finance. It includes assumptions on how the investors (agents) process the available information to form their expectations.

There are some firm points in the theory of rationality and information, which is scarce in the markets, so it is important to keep track of the relevant knowledge. Investors form their expectations in a way that depends on the structure of the economic system and the information history embedded in market prices ([Blavatsky, 2008](#)).

In finance, the outcome of many situations depends on the expectations of investors. Often, prices depend on the trading decision of the investors, which in turn depend on how the investors behave on the market.

People tend to rush in selling and buying assets, following the predictions they have on the market price. Financial markets tend to adjust very quickly to new information and investors must be quick in implementing their strategies.

There is a two-way flow of influences between expectations and outcomes, so people try to make forecasts of what will occur and base their expectations on that.

Better forecasting means better profits.

Investors adjust their expectations, and the way they interpret the information is highly dependent on the past outcome of the process. They adapt the forecasting rules to previous errors to eliminate them.

Rational expectations theory states that outcomes do not differ from people's predictions when they are based on rational processing of universally available information. Singularly taken, investors can make mistakes, but the various sides of the market will adjust rapidly and those mistakes will not be persistent.

The theory is based on the assumption that people behave to maximize their utility, and this belief is the core of a theory that tries to predict future outcomes of an economic system based on past decisions of the agents.

Rational expectations are at the basis of many theories, such as the random walk theory of financial assets, the efficient market hypothesis (EMH), economic theories of consumption and public economic policies.

The EMH applies rational expectations to efficient markets and asset pricing. It concludes that after adjusting for discounting and dividends, the changes in a stock price follow a random walk process.

The main bug in classical consumer choice theory is that all the results are drawn in a framework of certainty. However, as mentioned above, the real world is characterized by uncertainty, so bad things may happen, and agents must adapt to it.

Investors make choices in a context of uncertainty, and the outcomes are often unpredictable. However, there is a need to look forward and make predictions somehow. To obtain a realistic model of choice, it is necessary to model uncertainty.

A standard gamble has the following expected payoff:

$$E(x) = p_+ x_+ (1 - p_+) x_-$$

where

p_+ is the probability of a positive outcome.

x_+ is the positive outcome.

x_- is the negative outcome.

If asked about entering a fair gamble with a positive payoff, most people behave in such a way that they would reject a gamble even if it has an expected positive payoff when the uncertain prospects are worth less in utility terms than certain ones, even when expected tangible payoffs are the same.

Example 1.2: Assume you are offered a fair gamble. A coin is tossed with a positive payoff (win) of €1,000 if it is head and a negative payoff (loss) of €850 if it is tail. The expected value is

$$E(x) = 0.5 \times 1,000 - 0.5 \times 850 = \text{€}75$$

A positive value suggests that the gamble is worth accepting. However, the downside potential loss is so consistent, compared to the winning amount, that most people would reject the gamble.

To mathematically characterize the utility maximization framework, leading to the definition of risk and risk measurement, define a world with $1, 2, \dots, n$ possible states associated with probabilities $p_i = p_1, p_2, \dots, p_n$. The expected value is defined as

$$E(x) = \sum_{i=1}^n p_i x_i$$

Dispersion (variance) is measured as

$$\text{Var}(x) = \sum_{i=1}^n p_i (x_i - E(x))^2$$

Example 1.3: A stock has a 25% probability, in one year, of being worth €400 and a 75% probability of being worth €200. The expected value is

$$E(x) = 0.25 \times 400 + 0.75 \times 200 = \text{€}250$$

and the variance is

$$\text{Var}(x) = 0.25 \times (400 - 250)^2 + 0.75 \times (200 - 250)^2 = 7,500$$

By standard theory of utility, a utility function on the real domain $U|_{\mathbb{R}}$ has an expected utility form if it is possible to assign values u_1, u_2, \dots, u_n to the outcomes of a simple lottery. The expected value of the lottery is then given by

$$E(x) = \sum_{i=1}^n p_i u_i$$

where p_i is the probability of outcome i in the simple lottery.

Consider also the compound lottery defined as $(l_1, l_2, \dots, l_J; \pi_1, \pi_2, \dots, \pi_J)$ being the set yielding the lottery l_j with probability π_j . A utility function has the expected utility form if and only if

$$u\left(\sum_{j=1}^J \pi_j l_j\right) = \sum_{j=1}^J \pi_j u(l_j)$$

The shape of the utility function determines the different relationships between expected outcomes and the utility they give. In particular, for a lottery with n outcomes, the main distinction is between the expected value of the utility, defined as

$$E[u(x)] = \sum_{i=1}^n p_i u(x_i)$$

and the utility of the expected outcome, which is given by

$$u[E(x)] = u\left(\sum_{i=1}^n p_i x_i\right)$$

The risk premium involved in the choice is defined as the difference between the wealth after entering the gamble and a certain amount.

Example 1.4: Suppose an investor with an initial wealth $w = €100$ and utility function $U(w) = \sqrt{w}$. The risk premium associated with a gamble with a 50% probability of obtaining a wealth of €120 and 50% of lowering the wealth to €80 is given by calculating the expected utility first, as

$$E[u(w)] = 0.5\sqrt{120} + 0.5\sqrt{80} = 9.95$$

Since $u(w) = \sqrt{w} \Rightarrow w = [u(w)]^2$, the wealth associated with it is given by

$$w = 9.95^2 = €98.99$$

The risk premium is given by

$$RP = 100 - 98.99 = €1.01$$

The relationship between the expected utility and the utility of the expectation determines the risk attitude of the investors. Thus, if

$$E[u(x)] = u[E(x)]$$

where

$u(\cdot)$ is a concave utility function,

the investor is risk neutral if

$$E[u(x)] > u[E(x)]$$

the investor is risk-taker, and if

$$E[u(x)] < u[E(x)]$$

the investor is risk-adverse.