OVERLAPPING GENERATIONS

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OVERLAPPING GENERATIONS: METHODS, MODELS AND MORPHOLOGY

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To our wives, Ann and Sara
Who put up with all the time that went into this work.

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INTRODUCTION

In the early days of personal computing, there were two competing computer operating systems vying for consumers' dollars. One was Microsoft's MS-DOS system, and the other was Apple's MacIntosh system.

Microsoft's system was a clunky command line system, based on the original IBM disk operating system. Apple's system used a windowed graphical user interface based on Xerox's original X-windows interface for Unix. Of these two systems, the MacIntosh was by far the easier to use. Switching between applications was as simple as mousing the pointer from one window to another and clicking. On a Microsoft machine, switching involved quitting out one application, entering commands to start a new application, and then settling down to work in the new application, hopefully without needing to go back to the first one!

It is well-known by now that of these two competing computer operating systems, Microsoft became dominant, despite its obvious inferiority to the MacIntosh. The reason for this is the stuff of an Industrial Organization course case study, but the factors leading to Microsoft's dominance include the state of technology, political and economic differences of opinion, and the lock-in effects generated by the network of applications that ran on the two different platforms. That the MacIntosh was the better technology, though, is clearly apparent from the fact that MS-DOS steadily evolved from its initial command line structure into a system centered on Xerox's windowed graphical user interface, which Microsoft now calls Windows.

This particular example of what we call the standardization trap is not new. A careful study of the history of the automobile clearly shows that electric cars developed alongside steam- and gasoline-powered cars, with electric vehicles being the better technology in the early phases of development. The market for automobiles tipped to internal combustion only after Henry Ford's improvement on Ransom Olds' patented assembly line process allowed him to mass produce cheap Model T's that ran on gasoline. The superiority of electric vehicles is only now becoming apparent, with the advent of serious repercussions due to global warming, and with the development of high-capacity batteries that allow electric cars to travel long distances. But even before these recent developments, diesel-electric hybrid technology replaced steam as the optimal power source for heavy

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freight trains, and this technology has recently found its way into automobiles as well.

In the history of electrification, competing systems developed by Thomas Edison – direct current – and Nicola Tesla – alternating current – existed side-by-side, with alternating current finally winning out for the almost trivial reason that it is easier to change voltages with AC. Today, DC adapters routinely make voltage adjustments, and, as a result, all modern electronics run on DC. As locally generated electricity from renewable sources starts to come online, the electricity market might well tip back to Edison's favored DC technology.

Standardization traps aren't limited to new technological innovations. They occur in the realm of ideas as well. In fact, the scientific method operates by allowing competing hypotheses to be tested against empirical facts, with the hypothesis best able to explain the data enjoying at least temporary status as king of the hill. How long different hypotheses can stay on top depends on the quality of the empirical data available for testing, and on the sharpness of predictions, the reigning hypothesis generates.

Science characterizes reigning hypotheses as paradigms, although one should note that the original meaning of the word paradigm from Greek was one of side-by-side comparison. Thomas Kuhn originated the notion of a paradigm shift for the replacement or selection of one theory or hypothesis over others as the premier object of study.

In contemporary physics, we see this process at work in the parallel development of different theories for unifying general relativity's characterization of gravity with the underlying structure of quantum mechanics. As yet, there has been no compelling reason for one approach to supplant others, so no standardization trap or paradigm shift has occurred.

Historically, the debate over whether the universe was static or dynamic allowed both perspectives equal validity, to the point that Einstein made his famous blunder in setting the cosmological constant in the field equations of general relativity so that expansive and contractive forces would balance out, leaving the universe static. Despite the fact that in 1922, Alexander Friedmann had shown that the general relativity equations could accommodate different cosmological shapes and dynamics. Only in 1929, after Edwin Hubble discovered the uniform red shift of galaxies in all directions (with the implication that space was itself expanding), did Einstein correct his blunder. With this correction, a paradigm shift occurred, and the Big Bang became the standardized theory of cosmology. Whether a new theory will ultimately replace the Big Bang (and hence whether this was a trap) remains to be seen.

In the social sciences, a much slower research cycle, a relative paucity of empirical data, and limited possibilities for conducting experiments that generate new data lead to the possibility of much longer coexistence between competing theories, and greater opportunities for incomplete or incorrect theories to get locked in as the dominant paradigm, with all the attendant problems this raises for correcting the science when resources flow elsewhere.

We will argue here in this book that this is precisely the situation that macroeconomics and general equilibrium theory currently face. The problem arises out of the fact that two competing dynamic, stochastic, general equilibrium (DSGE) models have evolved as workhorse models for macroeconomic analysis, which we define broadly to include monetary economics, business cycle theory, economic growth, public finance/optimal taxation, and fiscal policy analysis.

To indicate the conundrum macro faces with these models, let us start with the crudest possible characterization of them.

The first model consists of a single, so-called "representative" agent who lives forever. Time enters the model as a sequence of discrete periods. The agent has access to technology for producing goods and services. The agent values these goods and services according to a utility function defined over consumption in each period, and the agent discounts utility in each period using a constant discount factor $0 < \beta < 1$. With these features in place, the agent's economic problem is to choose a series of investments in capital (or technology more generally) to maximize intertemporal utility subject to the constraint that invested resources can't be consumed.

The second model consists of a sequence of agents born at different times (with time also entering discretely) who overlap with and can trade with agents born earlier. Agents in this model live finite lifetimes. As with the first model, agents have access to technology for producing goods and services, and value consumption of goods and services via a utility function defined over consumption in each period of life. Agents in this model may or may not discount future utility. With these features in place, agents in this second model choose lifetime consumption and saving to maximize lifetime intertemporal utility, with savings becoming available for investment in technology.

By way of simplifying nomenclature, we will follow the conventional economic practice of referring to the first model as the infinite-lived agent (ILA) model and the second model as the overlapping generations (OLG) model.

The ILA model was originally developed by Frank Ramsey in 1928 as a way of modeling economic growth. Ramsey viewed the optimization problem posited in the model as the goal of a central planner, with the objective function of the problem (the representative agent's intertemporal utility) viewed as a social welfare function that put specified welfare weights on future generations' utility. The model was adopted and elaborated on by John Von Neumann (1937) and saw parallel development in the 1940s and 1950s by Maurice Allais (1947), Edmond Malinvaud (1953), and Robert Solow and Paul Samuelson (1956).

Work by David Cass in the mid-1960s showed how to completely characterize the optimal trajectories in this infinite-horizon general equilibrium model. Since Cass' work coincided with work by the then more senior and better-known Tjalling Koopmans, the resulting model is now universally known within the economics profession as the Ramsey–Cass–Koopmans (RCK) model, though it is sometimes also referred to as the neoclassical growth model. For details on these developments, see Spear and Young (2014).

The OLG model first appeared in 1947 in the Technical Appendix of Maurice Allais' book *Economie et Interet*. Because the model appeared only in the Appendix and the work in which it appeared was published in French, Allais' application of the model received very little attention in the English-speaking sector of the economics profession.

The OLG model was reinvented (or rediscovered) a decade later by Paul Samuelson (1958). Samuelson's model generated intense interest among general equilibrium theorists and monetary economists. General equilibrium theorists were interested in the fact that Samuelson's model could admit competitive equilibria which were not Pareto optimal, a contradiction of the First Welfare theorem for static general equilibrium models. Monetary economists focused on the fact that a role for money appeared naturally in Samuelson's model as a means of saving for retirement. A few years later, Peter Diamond (1965) reworked the OLG approach by introducing production into the model. This made possible a more realistic way of dealing with government debt and extended OLG into the areas of public finance, fiscal policy, and social security.

At roughly the same time that Samuelson was working on his early version of the OLG model, Gerard Debreu (1954) published work showing that Ramseyan-type models satisfied the second welfare theorem, allowing him to explicitly find prices that would support the Pareto optimal allocations obtained by the central planner as competitive equilibrium allocations for a large number of identical agents whose preferences were given by the central planner's social welfare function. This result was generalized by Takashi Negishi (1960) who showed how to extend Debreu's result to allow for any finite number of infinite lived agents. This appears to be the first instance in which the central planner was reinterpreted as an actual economic actor.

Now, on its face, a model in which actual economic agents live forever is problematic, and it isn't surprising that while there was some work being done on Samuelson's model in the 20 years after his seminal paper, there was no work (per a search of EconLit for "representative agent") being done on ILA models (as distinct from the planning version of the RCK model) during the comparable period.

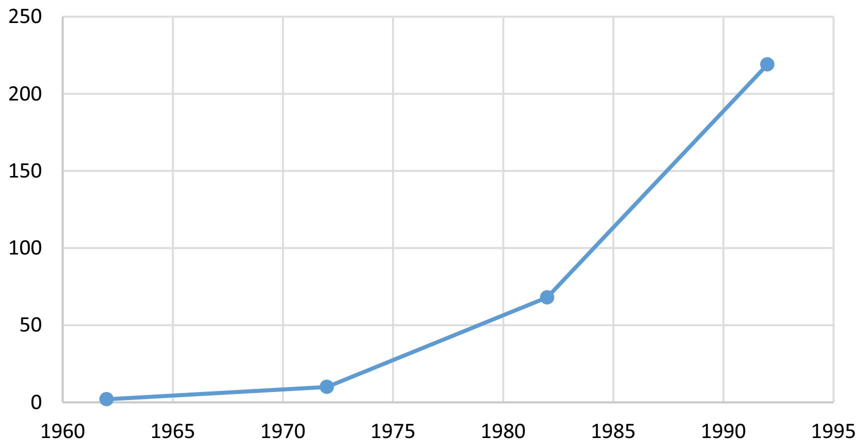
It is also not surprising that perhaps the most important paper in modern macroeconomics, Robert Lucas' 1972 paper "Expectations and the Neutrality of Money," was based on a very sophisticated stochastic extension of the OLG model. The OLG framework allowed (as we noted above) the introduction of money, and individual agents (as opposed to a planner), together with random shocks to both the demand for goods and the supply of money, which then allowed Lucas to present a simple model of how a signal extraction problem could generate a short-run positive relationship between inflation and real output. He also introduced macroeconomists and general equilibrium theorists to the rational expectations paradigm along the way.

Shortly after Lucas published his path-breaking '72 paper, Robert Barro (1974) took Samuelson's OLG framework and showed how it could be transformed into the representative agent ILA model of Debreu and Negishi. Barro's insight was that if one interpreted the finite-lived agents of the OLG model as families, and families propagated lineally, and each parent family left positive bequests to their immediate offspring such that the resulting intergenerational allocation maximized the discounted infinite sum of utilities of all generations, then the OLG model with bequests was identical to the representative agent ILA model. As an added benefit, Barro's construction showed how forward dynamic programming techniques introduced in economics a decade earlier by Roy Radner (1967) could be applied to indicate the generational separation between agents in the ILA model and to solve the infinite-horizon optimization without transversality conditions.

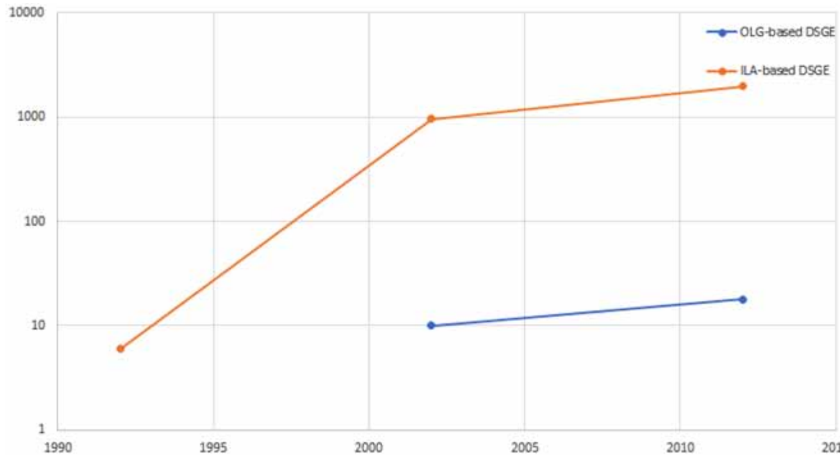
Barro's reinterpretation of OLG came at roughly the same time that William Brock and Leonard Mirman were showing how to extend the RCK model to stochastic environments, a development that then fed directly into the work of Finn Kydland and Edward Prescott which launched real business cycle theory – based on the stochastic representative agent ILA model – in the mid-1980s. For details on this history, see Young (2014).

The chart below, based on a search of the American Economics Association's EconLit database, shows the growth of publications based on the OLG model from its introduction through the early 1990s.

Viewed in isolation, this seems like impressive growth in interest in and work on OLG models. However, when we do an EconLit comparison of the DSGE research (i.e., using the search term "dynamic stochastic general equilibrium") using models based on the OLG framework with those based on the ILA framework, the result is stunning. Prior to the early 1990s, there is no research of any type on DSGE. The topic then explodes for ILA-based models, with some growth in the early 2000s for OLG-based models, as illustrated in the chart below.



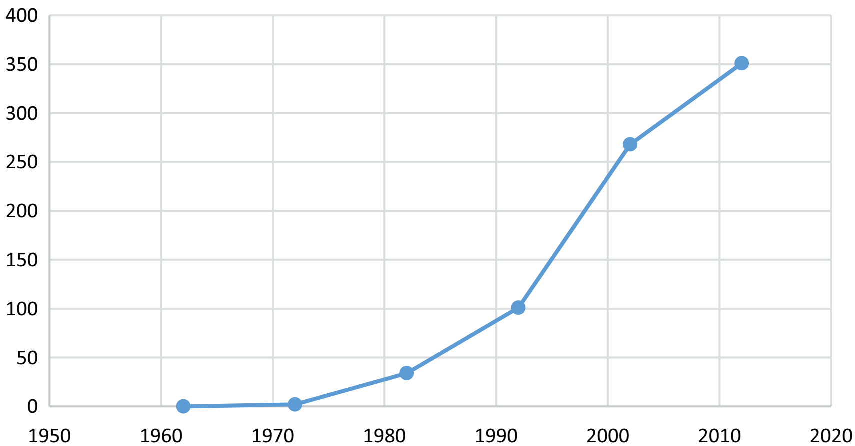
Early OLG Publications. *Source*: Compiled by authors from *Econlit* database, 1960–1995.



DSGE Research by Model Type. *Source*: Compiled by authors from the *Econlit* database, 1990–2015.

We use a log scale for this chart because the growth in ILA-based DSGE papers has been exponentially greater than that for OLG-based models.

While macroeconomists routinely describe OLG and ILA models as workhorses, it is clear that the ILA model is pulling a great deal more weight than OLG. At the same time, a search on "stochastic" and "overlapping generations"[1] generates the following:



Stochastic OLG. *Source*: Compiled by authors from the *Econlit* database, 1950–2020.

So, work on stochastic OLG models did continue apace – much of it stimulated by the publication of Lucas's '72 paper – and in fact grew, but at nowhere near the rate at which applications of the ILA model in macro were growing. Indeed, as we will see, even Lucas, who pioneered the use of stochastic OLG models, changed workhorses in midstream.

An additional piece of data that supports this contention comes from a study by Kim et al. (2006). Based on a list of papers appearing in major economics journals over the period 1970–2005 with at least 500 citations in the ISI Web of Science/Social Science Citation Index, the authors concluded that there had been a significant shift in the emphasis placed on theory as against empirical work, and on the various subfields that attracted the attention of researchers in economics, with a shift from both micro and macroeconomics to growth and development during the 1990s.

What is relevant for our purpose is the placement of four watershed papers – three based upon the dynastic/infinite-lived agent model, as against a single paper based on the overlapping generations model – in their listing of articles receiving more than 500 citations over the period. The three ILA papers were those by Barro (1974), Kydland and Prescott (1982), and Lucas (1988). The OLG-based paper is Lucas (1972). The 1988 *Journal of Monetary Economics* paper by Lucas is listed at number 15, with 1,772 citations. The 1974 *Journal of Political Economy* paper by Barro is listed at number 29, with some 1,209 citations. The 1982 *Econometrica* paper by Kydland and Prescott had 814 citations, coming in at 61. The OLG paper by Lucas, published a decade earlier, in the *Journal of Economic Theory* in 1972, had 838 citations, and comes in only three places before Kydland and Prescott, at number 58. Although published a decade after Lucas's OLG-based paper, the ILA-based Kydland and Prescott paper had caught up in terms of citation impact with that of Lucas. Lucas' own 1988 paper "On the Mechanics of Economic Development," which uses the ILA methodology, had also clearly eclipsed his earlier OLG paper.

This illustrates the twilight of the old OLG-based approach as proxied by Lucas (1972) and the rise of the new ILA-based approach as proxied by Kydland and Prescott (1982) and Lucas' own adoption of the ILA framework as his basic research agenda, albeit retaining OLG as an analytical device to be applied in specific cases; see, for example, Lucas (1986).[2] But more is at stake here. The Kim et al. piece clearly shows the shift away from theory toward empiricism, and the high placement of Barro's 1974 paper likely reflects the way macro people often started by positing an OLG model, but then allowed bequests and invoked Barro to justify working with the more convenient, recursive methodology provided by the ILA framework.

This is exactly the process we are trying to describe. The key question is why did macroeconomists feel the need to do this? Computational efficacy? Mathematical tractability? Ideological predisposition? If so, it illustrates the process of a particular paradigm coming to "rule the roost." And the fact that – with the exception of the 1972 paper by Lucas – there was nothing on the Kim et. al. list that reflects OLG, shows exactly what occurred. ILA came to rule the roost, not perhaps

Total Dissertations Awarded.

	OLG	CL	RCK	RBC	Stochastic Growth
1960–1979	2	8			84
1980–1999	522	128	12	407	491
2000–2020	1,254	133	49	1,394	639
Totals	1,778	269	61	1,801	1,214
OLG Total	2,047				
ILA Total	3,076				

Source: Compiled by authors from ProQuest Dissertations Database.

because it was the "best" or most realistic model, but rather because it was the easiest to work with.

Our final data showing the relative ascendence of the ILA model over OLG comes from the ProQuest database of dissertations awarded between 1960 and 2020. When we search by various model-related keywords – "overlapping generations," "consumption loans," "Ramsey-Cass-Koopmans," "real business cycles," and "growth" – we get the numbers reported in the table above.

As with the data on publications based on each model, while there has been growth in the number of doctoral students trained in both models, ILA outpaces OLG significantly.

The ILA applications in macro were not without critics and substantive critiques. Among the early critiques was a paper by Kyle Bagwell and B. Douglas Bernheim which made the obvious point that families don't propagate lineally. Rather, given standard taboos against incest, families propagate as trees. Bernheim and Bagwell (1988) showed that if family structures were modeled realistically, and all families were interconnected via positive transfers (as in Barro's model), then everything was neutral. Markets played no role in the Bernheim–Bagwell model for the simple reason that internal transfer adjustments between families could restore the optimality of allocations without the need for market interventions following any perturbations to the original optimal equilibrium. A more recent study by Engineer and Welling (2004) argues persuasively that the overlapping generations structure and the economic consequences generated by age heterogeneity conform closely to anthropological structures delineated in Stewart (1977) known as graded age-group sets. As Engineer and Welling argue, their results

> establish that OLG models can accurately represent a large subset of actual age-group societies called graded age-set societies. Thus, an OLG model bears a close resemblance to reality. (Engineer & Welling, 2004, p. 454)

Together with data showing that most families do not leave substantial bequests to their children – investing instead in human capital bequests via K-12 education and beyond – it is hard not to come away from a study of these two workhorse models without the feeling that one of them is a PC and the other is a Mac.

Our purpose here, then, is to elucidate the technical and intellectual forces that have generated the current situation in which macroeconomics seems to be caught in a standardization trap based on a problematic choice of models.