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Understanding and Designing Structures without a Computer

Spatial Structural Systems, Dynamics and Foundations

Leonidas Stavridis and
Konstantinos Georgiadis

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Contents

Foreword

Methods of structural analysis have experienced an explosive growth during the last 40 years. But it was the advent of powerful personal computers, along with the evolution of numerical tools (based mainly on the finite element method) and the parallel development of numerous reliable, comprehensive, commercially available computer software, that have enabled engineers to tackle very complex structural systems. As a consequence, in today's design offices, analysis of even some rather simple systems is performed (especially by the younger generation of engineers) with the use of such computer codes. Classical as well as modern methods of structural analysis (based on the principles of virtual work, compatibility of deformations, matrix analysis) are rather rarely invoked in everyday practice. Yet, these theoretical tools often constitute the major (if not the only) part of the curriculum in civil engineering schools.

Several problems may arise from this state of affairs. First, the danger of the 'black-box syndrome': when a sophisticated code is used without the analyst having the ability to check whether the results are indeed reasonable and to spot any errors in the physical meaning of their implicit assumptions and how these assumptions are materialised in the model. Second, there is little if any training to help the young engineer develop a deeper understanding of how structural systems behave, let alone to sharpen their physical intuition; such understanding and intuition are necessary especially in the conception and preliminary design stages. Indeed, conceptual clarity and physical insight are rarely mentioned as key objectives of structural analysis courses.

This two-volume book by Professor Leonidas Stavridis and Dr Konstantinos Georgiadis offers a much-needed addition to classical computational structural analysis. A physical approach is developed in which a structural system is decomposed into elements whose behaviour to the applied loads is easily computed 'from the basics'. Starting in the first chapters with fundamental concepts and applications, the step-by-step exposition becomes progressively more advanced. Structural analysis blends naturally with mechanics of materials – the latter include reinforced and prestressed concrete, steel and composites. The in-depth analysis of standard structural systems (such as simply and multi-supported beams, frames, arches, cabled beams) is followed by the exposition of some more advanced topics such as buckling, slabs and shells, thin-walled and box girders, grids and curved beams, laterally loaded multi-storey frames and shear walls.

It is amazing how the analysis of such complex systems is made so simple, clearly understandable even to a non-specialist civil engineer, as the present writer. This is accomplished to a large extent thanks to the numerous illustrative figures (sketches) that go far beyond the usual 'formalistic' figures of most available textbooks: they are imaginative, vivid, self-explanatory. What a difference they make when trying to comprehend difficult topics! For instance, the chapter on ‘Shells’ contains 56 elaborate figures, most of which comprise several sketches while a few of them are a whole page long. The three-dimensional nature of cylindrical, spherical, paraboloid and conical shells is elucidated with the help of ingeniously selected isometric views and numerous cross-sections so that the reader feels that this is a rather simple subject.

As an engineer with a special interest in soil–foundation–structure interaction, I was particularly happy with the comprehensive treatment of foundations. Viewed mainly from a structural engineer’s viewpoint, the pertinent chapter deals not only with some classical deformation–settlement and stress–distribution problems, but also with the interplay between foundation stiffness and structure distress.

I believe this book will prove invaluable to both students and practising engineers in helping them not only to absorb a huge volume of material, but also (more significantly) to cultivate ‘engineering intuition’ and develop insight into the physics of structural analysis. For students, in particular, all this will offer the motivation for further study and the desire to later apply in real-life projects both the material and the methodology developed in the book.

George Gazetas, Professor of Soil Mechanics and Foundation Engineering, National Technical University of Athens

Preface

Πεπαιδευμένον γαρ εστίν επί τοσούτω το ακριβές επιζητείν όσον η του πράγματος φύσις επιδέχεται.

Αριστοτέλης

Because it is the essence of education to seek as much accuracy as the nature of things allows.

Aristoteles

A technically educated person – whether an engineer, architect or builder – today understands 'structural design' in much the same way as their predecessors did 500 years ago: as a practical procedure that applies specialised knowledge to ensure a structure 'stands up' and 'does not fall down,' resisting whatever loads it encounters during its lifespan.

Yet, what has evolved across the centuries – transforming structural theory from empiricism into a rigorous scientific discipline – is the introduction of analytical methods. The capacity to assess structural behaviour systematically, and the advancement of computer-based methods and tools, have fundamentally shaped this discipline. Structural mechanics is now a highly demanding subject, spanning both the analytical evaluation of structural behaviour and its practical application in design. Though intimately related, these two realms retain distinct focuses.

Analytically, the central question is: Given a particular structural system and specified loads, what are its resulting forces and deformations? Answering this requires a strict scientific approach – one that can, at times, give the impression that the analysis itself is the end goal. Indeed, modern computing methods and software have made these calculations routine.

On the other hand, practical design emphasises the art of applying this understanding to create efficient load-carrying solutions that are economical, functional and visually pleasing. Given a particular set of service requirements and environmental conditions, what structural concept – using appropriate materials – will best satisfy the design criteria? This is where engineering insight and creativity matter most.

Although engineering curricula tend to focus on the scientific side, aspiring structural engineers often discover too late that true mastery requires more than rote reliance on computer analysis or prescriptive codes. Equally important is the ability to 'see' how forces flow through a structure – to perceive its behaviour as a coherent system. Without this intuitive understanding, engineers may find themselves ill-equipped to engage meaningfully with architects and builders on real-world projects.

This two-volume book aims to bridge that gap. It explores the behaviour of a wide range of structural systems – beams, frames, arches, cables, grillages, slabs, shells, thin-walled sections and multi-storey structures – placing particular emphasis on the underlying mechanisms that allow them to support loads. It discusses traditional materials like steel and concrete alongside composite and prestressed solutions, and introduces the principles of plastic analysis, second-order theory and structural stability in a simplified manner.

Special chapters address the design of statically determinate and indeterminate plane structures, dynamic response under seismic and human-induced actions, and the treatment of shallow and deep foundations – recognising that structural design is never complete without an understanding of soil–structure interaction.

The book adopts a progressive, concept-building approach: each chapter builds on earlier material, ensuring readers establish a strong intuitive and analytical base before moving on to more advanced topics. Some background knowledge of elementary mechanics is assumed.

As Vitruvius wrote more than two millennia ago, successful structural design must satisfy four core criteria: safety, functionality, economy and beauty. Technical safety requires that a structure's capacity exceed the demand; functionality requires limiting displacements and vibrations; economy requires choosing efficient structural forms and construction methods; and beauty requires sensitivity to proportion and elegance. Achieving these ideals depends as much on an engineer's creativity and judgement as on their technical prowess.

Ultimately, this book is for anyone – student, practising engineer or architect – who wishes to gain deeper structural insight. Our hope is that it will not only enrich readers' appreciation of structural behaviour, but also help them to design with greater understanding and confidence.

Finally, we wish to thank Emerald Publishing and its editorial team, led by Dr Michael Fenton, for their invaluable guidance and support in bringing this book to fruition.

Leonidas Stavridis

Konstantinos Georgiadis

About the authors

Professor Leonidas Stavridis obtained his Diploma in Civil Engineering and his PhD from the National Technical University of Athens (NTUA). Subsequently, he attended a postgraduate course in Bridge Engineering and Prestressed Structures at the Federal Institute of Technology of Zurich (ETH) where he obtained his diploma in 1989. In 2011, after more than 20 years of teaching structural analysis, bridge engineering and structural behaviour at both undergraduate and postgraduate level, he was elected Professor of Structural Engineering and Design at NTUA.

He is the author of *Structural Systems: Behaviour and Design* published in 2010 by ICE Publishing. In addition, his publications in peer-reviewed international journals cover a wide range of topics related to static and dynamic analyses of orthotropic slabs and shallow shells, prestressed cable structures, structural behaviour of multi-storey buildings, dynamic behaviour of curved thin-walled beams, soil–structure interaction problems including prestressed foundations, the treatment of external partial prestressing in slab bridges and the behaviour of suspension and stress ribbon.

Professor Stavridis has been an active freelance structural engineering consultant for more than 45 years and has been involved in various design projects, mostly in Greece, including multi-storey buildings, prestressed concrete bridges, space covering roofs and special foundations as well as structural restoration and strengthening of traditional monumental structures.

Dr Konstantinos Georgiadis is a Chartered Engineer (CEng MICE) with over 10 years of experience in bridge engineering and structural design. He graduated from the National Technical University of Athens (NTUA) in 2011 and earned his MSc in Steel Design and Business Management with distinction from Imperial College London in 2012.

He began his professional career in 2013, gaining experience in the UK railway sector with Bouygues. In 2014, he joined AECOM as a Bridge Engineer in the structures team. In 2016, he was awarded a prestigious EPSRC scholarship to pursue a PhD in Bridge Design at Imperial College London. During his doctoral studies, he served as a Teaching Assistant in Steel Design at Imperial and worked as a part-time external consultant for AECOM. After completing his PhD in 2020, he joined Arup in Hong Kong as a Chartered Senior Bridge Engineer and later transferred to their London office.

Dr Georgiadis has led and contributed to feasibility studies, preliminary and detailed designs, structural assessments and inspections of numerous bridges, including long-span structures such as cable-stayed and suspension bridges. His international project experience spans the UK, Hong Kong, Macau, the Philippines and Portugal. He is well-versed in multiple design codes, including the Eurocodes, British Standards and AASHTO.

With a strong technical background, Dr Georgiadis specialises in advanced linear and nonlinear, static and dynamic analysis and is a proficient user of various finite element analysis software. He also brings substantial site experience, having supervised bridge construction, inspections and surveys. He has authored multiple technical and research papers, which have been published and presented in leading journals and international conferences and has served as member of the Scientific Committee of IABSE.

Introduction

This book aims to cultivate a deep understanding of the structural behaviour and design principles of civil engineering structures. Its focus is on fundamental concepts concerning the design and analysis of various structural types, including different types of bridges, long-span roof structures and multi-storey buildings, among others. As the selection of the appropriate construction material plays an important role in the design, this book focuses on the main construction materials – that is, steel and concrete – as well as their combinations in the form of prestressed concrete and composite structures.

It covers the conceptual design of different structural forms, fostering a solid mindset to develop a sound preliminary design. Readers will gain an understanding of load-transferring paths and the ability to estimate the internal forces in structures – both qualitatively and quantitatively – without the need to rely on computer software or design codes. Of course, these tools will eventually be used when producing the detailed design and final construction drawings, which remain the ultimate goal of any structural project.

This second volume expands the discussion to include structures in space (i.e. three-dimensional structures) under vertically (e.g. gravity) or laterally applied loads (e.g. wind or seismic) in the form of large roofs, horizontal spanning systems or multi-storey buildings. In particular, the behaviour of the latter structures under dynamic seismic forces is explored through lumped-mass representations, highlighting the influence of material ductility on response and design.

This volume starts with the investigation of grillages, introducing the concept of torsion and its implications. This naturally leads to the examination of concrete slabs – including flat slabs and ribbed configurations – and the role of prestressing in enhancing their performance. Next, thin-shell structures are discussed, with an emphasis on their primary membrane action as well as the bending effects that may lead to localised stresses. Hypar shells receive particular attention due to their efficiency in construction, which is achieved through straight-line generatrices.

The behaviour of thin-walled beams and box-girder bridges is also covered, examining torsional effects, warping and the resulting axial stresses in open- and closed-sections. Multi-storey structures, with a focus on lateral load resistance and the interplay between horizontal and vertical elements – frames, shear walls and cores – are also presented.

The penultimate chapter focuses on the dynamic behaviour and seismic response of multi-storey buildings, with a particular emphasis on the role of material ductility in design loads. Vibrations induced by human activity or machinery are also discussed, as they often influence the serviceability of beams and plates.

The final chapter addresses mainly shallow foundations, presenting their design principles as well as basic soil–structure interaction concepts. Pile foundations are also briefly discussed.

Throughout this book, the underlying philosophy is that engineering judgement and structural insight is more important than computer analyses. Critical checks – informed by an understanding of the basic principles outlined here – always remain essential. Ultimately, every design process must lead to one final outcome: a well-conceived set of structural drawings that clearly communicate the configuration and construction method for building the project.

Leonidas Stavridis and Konstantinos Georgiadis
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Chapter 1
Grillages

1.1. Introduction to grillages and torsion

1.1.1 Equilibrium conditions

This section introduces the fundamental concepts related to the stress and deformation behaviour of grid structures, so-called *grillages*. In contrast with typical plane structures, which are loaded within their plane, grillages are plane structures that are loaded perpendicularly to their own plane, as shown in Figure 1.1, thus creating a spatial structural system.

For analytical purposes, a grillage is assumed to lie within a horizontal plane defined by two arbitrary axes, *OX* and *OY*, as part of a three-dimensional coordinate system *OXYZ*. For any cut-out part of a grillage, the following conditions should be satisfied for equilibrium:

$$\sum \overrightarrow{M_x} = 0, \qquad \sum \overrightarrow{M_y} = 0, \qquad \sum \overrightarrow{P_z} = 0$$

These expressions indicate that the sum of the moment vectors of all external forces about the *OX* and *OY* axes must be zero. In addition, the sum of all vertical (out-of-plane) forces acting on the considered portion of the grid must also equal zero.

1.1.2 Internal forces and deformations

To accurately describe the internal forces and deformations at a specific point along a grillage member, it is useful to consider a vertical plane to the member at the point of interest. The intersection of this plane with the member exposes two adjacent cross-sections (free edges). These edges exhibit identical deformation vectors, reflecting the continuity of the structure. However, the vectors of internal forces at these sections are equal in magnitude but opposite in direction, ensuring equilibrium within the member.

The internal forces at any point along a grillage member are most effectively described by referring to a local orthogonal coordinate system (x, y, z), as illustrated in Figure 1.2. This coordinate system is centred at the centroid of the member's cross-section, with the x-axis aligned with the member's longitudinal axis. The y- and z-axes lie in the plane of the cross-section: the y-axis oriented horizontally and the z-axis vertically. It is assumed that the cross-section is symmetric about the horizontal y-axis.

The internal forces and the corresponding deformations are expressed using vector notation. The moment and rotation vectors shown in Figure 1.2 adhere to the *screw rule*: the direction of each vector indicates the sense in which a right-hand-threaded screw would rotate to move in that direction. When a grillage member is subjected to loading within its vertical plane (x–z), the following internal forces are developed (see Figure 1.2):

Figure 1.1 Equilibrium conditions for a loading perpendicular to a plane structure (grillage)

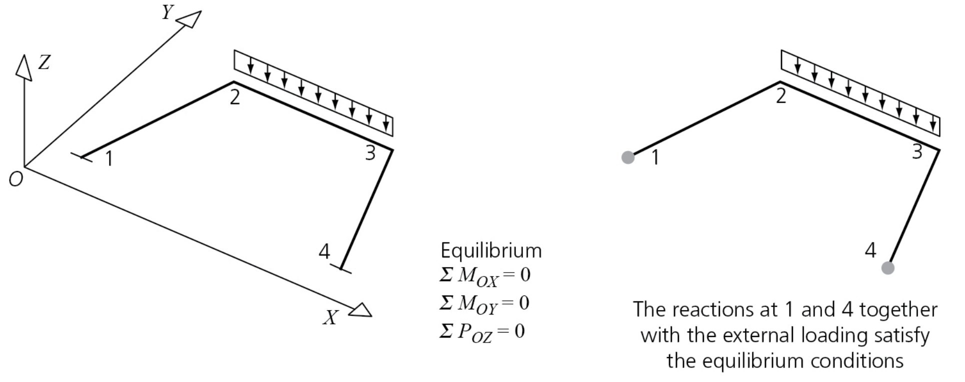
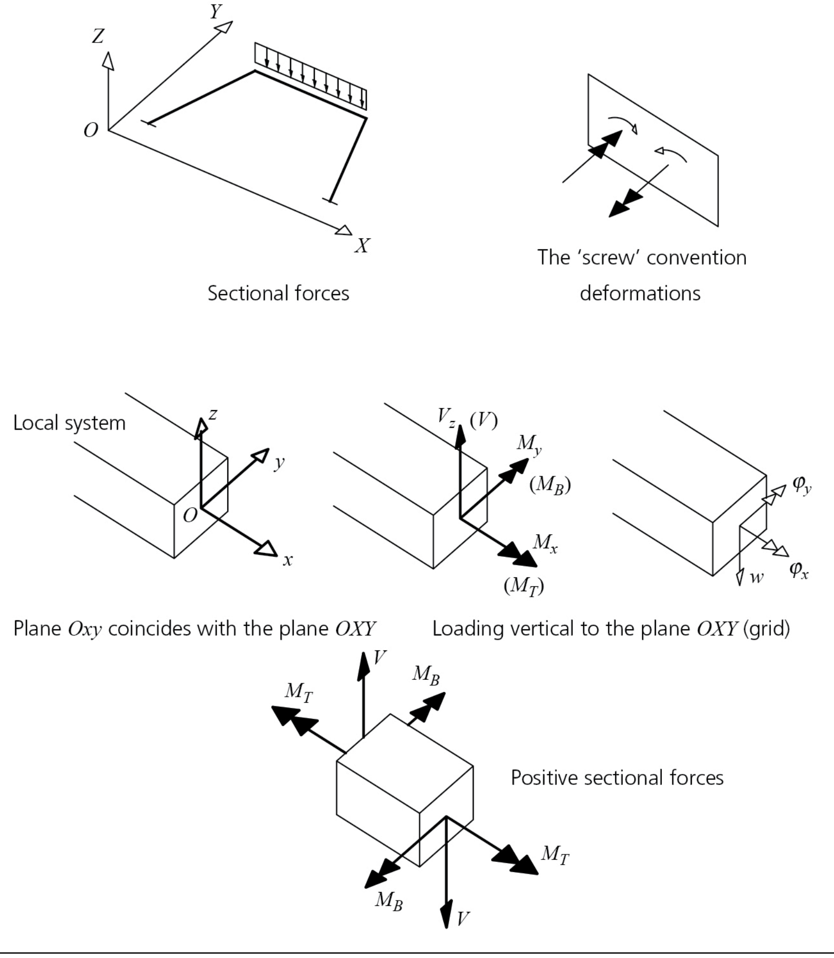


Figure 1.2 Internal forces and deformations developed in a grid structure



- shear force, V_z (i.e. V)
- bending moment, M_y (i.e. M_B)
- torsional moment, M_x (i.e. M_T).

The bending moment, M_B, in combination with the shear force, V, induces flexure in the vertical plane (x–z) of the member. The torsional moment, M_T, on the other hand, causes twisting about the member's longitudinal axis. At each point along the member, the resulting deformations include a vertical deflection, w, along the z-axis and a rotation represented by a horizontal vector. This rotation has two key components

- φ_y, which is the projection of the rotation vector onto the y-axis, associated with flexural (bending) rotation
- φ_x, which is the projection onto the x-axis, representing the torsional (twisting) angle.

The signs of the internal forces are defined as follows (see Figure 1.2).

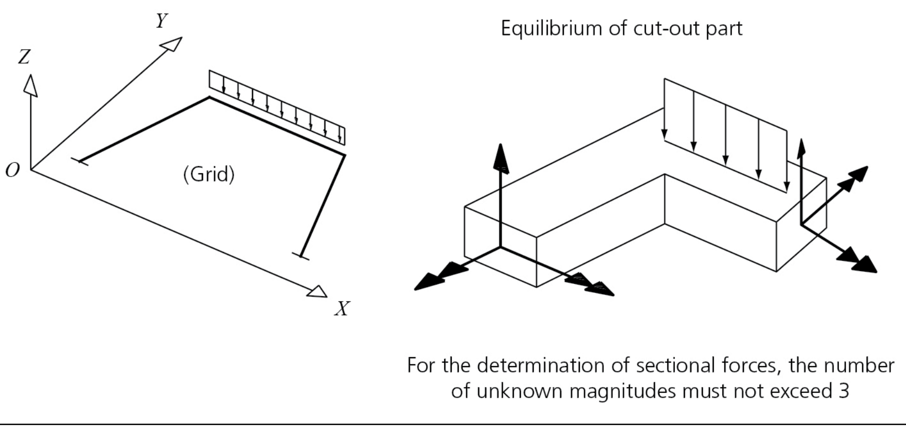
- *Shear force,* V, is positive when it, along with the corresponding force at an adjacent cross-section, produces a clockwise moment couple.
- *Bending moment,* M_B, is positive when it induces tension in the bottom fibres of the cross-section.
- *Torsional moment,* M_T, is positive when the rotation induced by its physical action on the cross-section results in a screw motion giving 'tension' for the cross-section itself.

These internal forces can be determined from the equilibrium conditions described previously, provided that all externally applied forces are known (see Figure 1.3).

For the determination of any deformation, the principle of virtual work is always applicable, which, in order to allow for the contribution of torsional moments in carrying the vertical loads, is now written in the following form:

$$\sum P^{virt} \cdot \delta^{resp,real} = \int M_B^{virt} \cdot \frac{M_B^{real}}{EI} \mathrm{d}s + \int M_T^{virt} \cdot \frac{M_T^{real}}{GI_T} \mathrm{d}s$$

Figure 1.3 Determination of the internal forces in a grid structure



where by the term $\int (M_T^{real} / GI_T)\, ds$ expresses the twisting angle $\Delta\varphi_{real}$, according to Section 1.1.4 in analogy with the discussion in Stavridis and Georgiadis (2025, Section 2.3.3).

1.1.3 Types of supports

There are two main ways of supporting a grillage structure (see Figure 1.4).

- *Simple support*: which simply prohibits vertical movement. The deformations φ_x and φ_y are freely developed. The only reaction developed is a vertical force, R.
- *Fixed support*: which restrains all possible deformations. The developed reactions are a vertical force, R, and the two moments M_x and M_y.

If the whole grillage is supported in such a way that only three reaction forces are developed, then the reactions can be determined from the three equilibrium conditions, as mentioned in Section 1.1.1. If more than three reaction forces exist, the equilibrium conditions are insufficient for the determination of these forces. The structure is statically indeterminate and the reaction forces can be determined following the methods discussed in Stavridis and Georgiadis (2025, Chapter 3) (i.e. the method of forces or the method of deformations).

1.1.4 Torsion

It is already clear that the internal force that differentiates grids from typical plane structures is torsion. For a member of length, L, the relation of the developed torsional moment, M_T, at its free end to the corresponding twisting angle, φ_T, of the end section (see Figure 1.5) is:

$$\varphi_T = M_T \cdot \frac{L}{GI_T}$$

where I_T is the *torsional moment of inertia* (which is a geometric property of a cross-section that measures its resistance to torsion (twisting) about its longitudinal axis); G is the shear modulus of the member's material, $G = \dfrac{E}{2(1+v)}$ and v is Poisson's ratio (see Stavridis and Georgiadis, 2025, Section 2.3.2).

For a rectangular cross-section having dimensions b/t ($b>t$), the torsional moment of inertia, I_T, is calculated according to the expression $I_T = b{\cdot}t^3/\eta$, where the coefficient η varies from 3.0 to 7.0 for oblong to square sections, respectively.

Figure 1.4 Two ways of supporting a grid structure

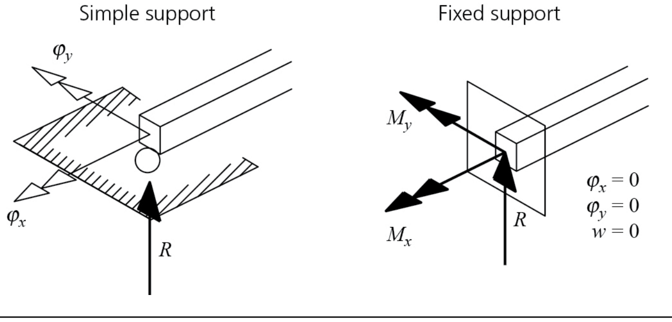
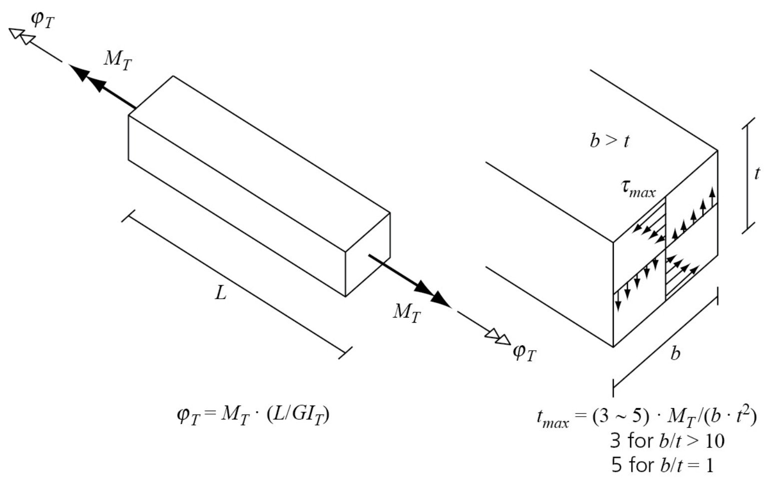


Figure 1.5 Torsional response of a beam with an orthogonal solid section



From the above equation of φ_T it can be concluded that the magnitude (GI_T/L) expresses the *torsional stiffness* of the member, which is the torsional moment required in order to produce a unit twisting angle or, in other words, the torsional resistance offered by the member if subjected to a unit twisting angle $\varphi_T = 1$.

A torsional moment, M_T, applied to a rectangular cross-section causes a flow of shear stresses on the full cross-section, which diminish towards the centre of the cross-section. The resultant moment vector with respect to the centre of the cross-section from these stresses is obviously identical to M_T (see Figure 1.5). The maximum shear stress, τ_{max}, appears at the middle of the longest side and is proportional to the stress magnitude $M_T/(b{\cdot}t^2)$ multiplied by a factor 3.0 for oblong sections and a factor 5.0 for square sections. It must be noted that τ_{max} is inversely proportional to the square of the thickness of the cross-section (see Figure 1.5).

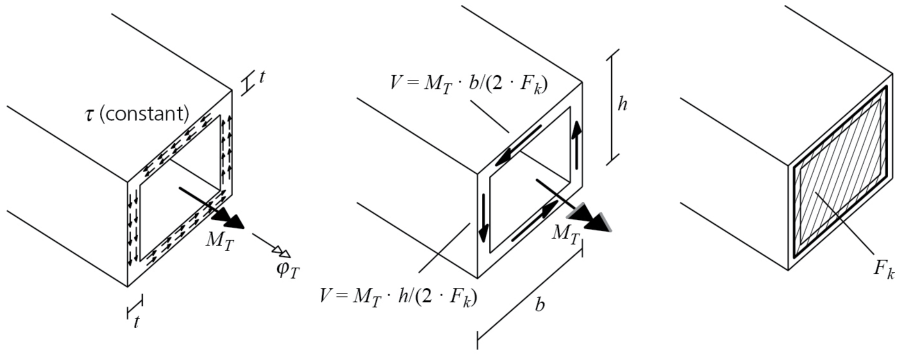
Apart from rectangular cross-sections, another common type of cross-section that is used in practice when torsion is present is the so-called *hollow box*, as shown in Figure 1.6. Each wall of the cross-section has a certain thickness, t, and takes up the torsional moment, M_T, through a peripheral constant shear stress, τ, which, according to the so-called *Bredt formula* is equal to:

$$\tau = \frac{M_T}{2 \cdot F_k \cdot t}$$

where F_k is the area enclosed by the middle line of the walls – that is, slightly less than the area of the full (solid) cross-section (see Figure 1.6).

Each wall of the hollow box cross-section is subjected to a constant shear flow $v = \tau{\cdot}t = M_T/(2{\cdot}F_k)$ acting per unit length (kN/m) of the wall, which thus causes a shear force $V = v{\cdot}l$ over the length, l, of each wall, where l is either h or b (see Figure 1.6). It can be seen from the equation of the shear flow, v, that the shear flow, v, corresponding to a certain torsional moment, M_T, remains constant,

Figure 1.6 Torsional response of a beam with hollow box section



The shear force of each wall is proportional to its height

even if the walls' thicknesses are different. It should be pointed out that this shear flow provides the whole section with the torsional moment ($2{\cdot}v{\cdot}F_k$), which is statically equivalent to the applied moment, M_T.

Of course, the previous relation between the twisting angle, φ_T, and the torsional moment, M_T, is also valid for hollow box cross-sections, the torsional moment of inertia, I_T, being now equal to $I_T = 4{\cdot}t{\cdot}F_k^2/L_\Omega$, where L_Ω is the perimeter of the middle line of the section. It is pointed out that the torsional moment of inertia I_T – and, consequently, the torsional stiffness (GI_T/L) – is, for the same cross-sectional area, much greater in a hollow than in a solid section.

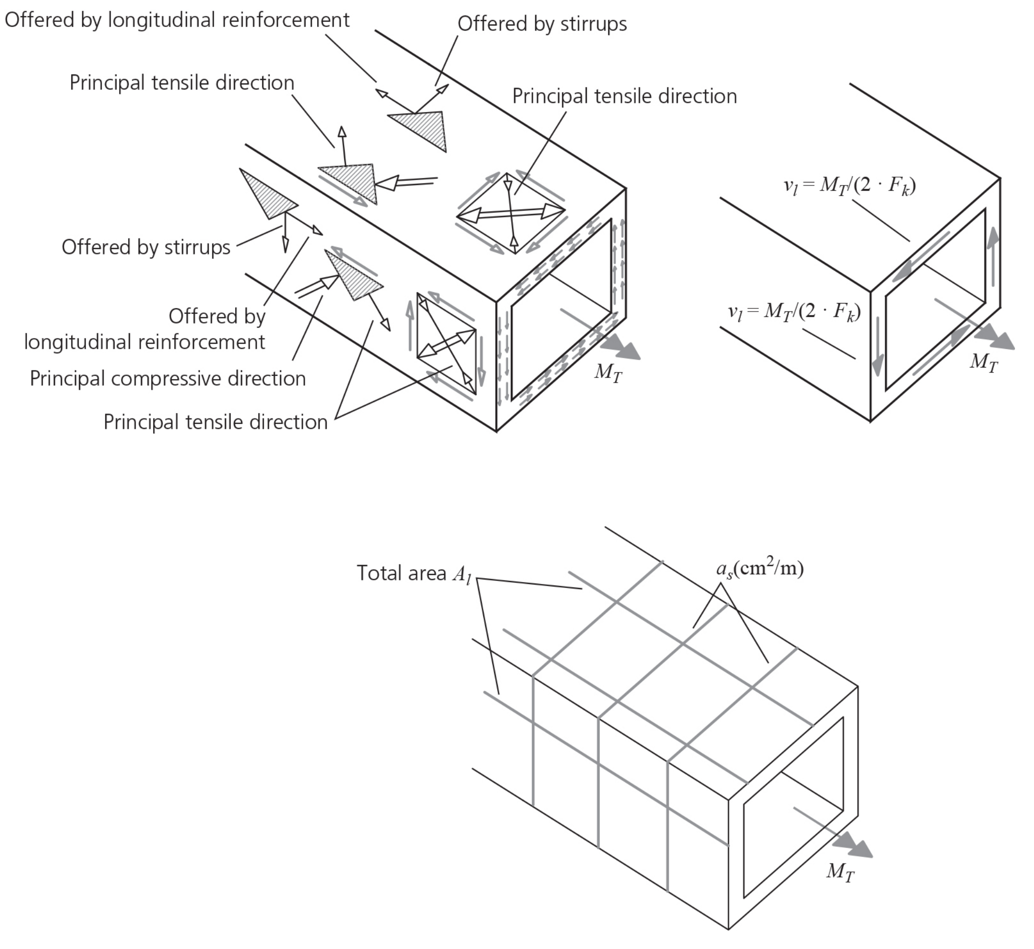
1.1.5 Design for torsion

As previously explained, in a rectangular cross-section subjected to torsion, the highest shear stresses are concentrated near the periphery of the section. This observation justifies modelling a solid section as a thin-walled hollow section, with the assumption that the entire torque is carried by a narrow region around its outer boundary of the section, as discussed in Section 1.1.4. This modelling approach is conservative as it results in higher stresses, thereby remaining on the safe side for design purposes. Consequently, the analysis of thin-walled hollow sections has broader applicability and practical significance in torsional design.

The shear flow developed around the thin wall that resists the applied torque gives rise to principal stresses, which are the actual physical stresses that are developed in the member (see Figure 1.7). The tensile principal stresses in steel sections can be resisted by the tensile capacity of the steel, but in concrete sections an appropriate reinforcement layout must be provided. This is typically achieved through the use of closed rectangular stirrups in combination with longitudinal reinforcement, which together provide the required oblique tensile force paths to resist the induced stresses (see Figure 1.7).

Allowing a service stress, σ_s, for the reinforcement steel results in a required section, a_s, per unit length for the stirrups equal to $a_s = v_l/\sigma_s$ (cm^2/m), and in a total cross-sectional area, A_l, for the longitudinal bars (uniformly distributed along the perimeter) equal to $A_l = (v_l/\sigma_s){\cdot}L_\Omega$.

Figure 1.7 Thin-walled model for torsional design of reinforced concrete members



Finally, it should be mentioned that, apart from grillages, torsion can also appear in typical plane structures, as presented in Stavridis and Georgiadis (2025). In any case, the torsional requirements (e.g. torsional reinforcement) must be considered in addition to the design requirement from other actions such as bending and shear.

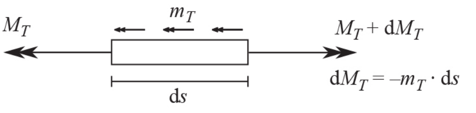
1.1.6 Static equilibrium and torsional deformation

If a beam of length, L, with fixed torsional support is subjected to a uniformly distributed torsional load, m_T, it develops a torsional moment diagram, M_T. Expressing the torsional equilibrium of an element of length, ds, of the beam, the following relation in plausible differential form is obtained (see Figure 1.8):

$$\frac{\mathrm{d}M_T}{\mathrm{d}s} = -m_T$$

The analogy of the above relationship with the relationship between a distributed load and the developed shear force, as discussed in Stavridis and Georgiadis (2025, Section 2.2.3), suggests

Figure 1.8 Equilibrium of an element of length, ds, of a fixed-ended beam under uniformly distributed torsional load, m_T



that the torsional moment diagram of the above beam is identical to the shear force diagram of an equivalent simply supported beam under a vertical uniformly distributed load equal to m_T (see Figure 1.9).

The same analogy is also valid for a concentrated torsional load applied at some point of the fixed-ended beam (see Figure 1.10).

Moreover, as the twisting angle, φ, actually changes along the beam in correlation with the corresponding torsional moment, M_T, it can be written in accordance with Section 1.1.3:

$$\frac{d\varphi}{ds} = \frac{M_T}{G \cdot I_T}$$

Regarding now the determination of the twisting angle diagram, a similar procedure to that given in Stavridis and Georgiadis (2025, Section 2.3.6) may be followed. By differentiating the above expression of dφ/ds, the following is obtained:

Figure 1.9 Analogy of a torsional beam under a uniformly distributed torsional load, m_T, with a simply supported beam with a vertical uniformly distributed load, $p(x)$