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Understanding and Designing Structures without a Computer

Plane Structural Systems

Leonidas Stavridis
Konstantinos Georgiadis

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Foreword

Methods of structural analysis have experienced an explosive growth during the last 40 years. But it was the advent of powerful personal computers, along with the evolution of numerical tools (based mainly on the finite element method) and the parallel development of numerous reliable, comprehensive, commercially available computer software, that have enabled engineers to tackle very complex structural systems. As a consequence, in today's design offices, analysis of even some rather simple systems is performed (especially by the younger generation of engineers) with the use of such computer codes. Classical as well as modern methods of structural analysis (based on the principles of virtual work, compatibility of deformations, matrix analysis) are rather rarely invoked in everyday practice. Yet, these theoretical tools often constitute the major (if not the only) part of the curriculum in civil engineering schools.

Several problems may arise from this state of affairs. First, the danger of the 'black-box syndrome': when a sophisticated code is used without the analyst having the ability to check whether the results are indeed reasonable and to spot any errors in the physical meaning of their implicit assumptions and how these assumptions are materialised in the model. Second, there is little if any training to help the young engineer develop a deeper understanding of how structural systems behave, let alone to sharpen their physical intuition; such understanding and intuition are necessary, especially in the conception and preliminary design stages. Indeed, conceptual clarity and physical insight are rarely mentioned as key objectives of structural analysis courses.

This two-volume book by Professor Leonidas Stavridis and Dr Konstantinos Georgiadis offers a much-needed addition to classical computational structural analysis. A physical approach is developed in which a structural system is decomposed into elements whose behaviour to the applied loads is easily computed 'from the basics'. Starting in the first chapters with fundamental concepts and applications, the step-by-step exposition becomes progressively more advanced. Structural analysis blends naturally with mechanics of materials – the latter include reinforced and prestressed concrete, steel and composites. The in-depth analysis of standard structural systems (such as simply and multi-supported beams, frames, arches, cabled beams) is followed by the exposition of some more advanced topics such as buckling, slabs and shells, thin-walled and box girders, grids and curved beams, laterally loaded multi-storey frames and shear walls.

It is amazing how the analysis of such complex systems is made so simple, clearly understandable even to a non-specialist civil engineer, as the present writer. This is accomplished to a large extent thanks to the numerous illustrative figures (sketches) that go far beyond the usual 'formalistic' figures of most available textbooks: they are imaginative, vivid, self-explanatory. What a difference they make when trying to comprehend difficult topics! For instance, the chapter on ‘Shells’ contains 56 elaborate figures, most of which comprise several sketches while a few of them are a whole page long. The three-dimensional nature of cylindrical, spherical, paraboloid and conical shells is elucidated with the help of ingeniously selected isometric views and numerous cross-sections so that the reader feels that this is a rather simple subject.

As an engineer with a special interest in soil–foundation–structure interaction, I was particularly happy with the comprehensive treatment of foundations. Viewed mainly from a structural engineer’s viewpoint, the pertinent chapter deals not only with some classical deformation–settlement and stress–distribution problems, but also with the interplay between foundation stiffness and structure distress.

I believe this book will prove invaluable to both students and practising engineers in helping them not only to absorb a huge volume of material, but also (more significantly) to cultivate ‘engineering intuition’ and develop insight into the physics of structural analysis. For students, in particular, all this will offer the motivation for further study and the desire to later apply in real-life projects both the material and the methodology developed in the book.

George Gazetas, Professor of Soil Mechanics and Foundation Engineering, National Technical University of Athens

Preface

> Πεπαιδευμένον γαρ εστίν επί τοσούτω το ακριβές επιζητείν όσον η του πράγματος φύσις επιδέχεται.
>
> Αριστοτέλης

> Because it is the essence of education to seek as much accuracy as the nature of things allows.
>
> Aristoteles

A technically educated person – whether an engineer, architect or builder – today understands ‘structural design’ in much the same way as their predecessors did 500 years ago: as a practical procedure that applies specialised knowledge to ensure a structure ‘stands up’ and ‘does not fall down’, resisting whatever loads it encounters during its lifespan.

Yet, what has evolved across the centuries – transforming structural theory from empiricism into a rigorous scientific discipline – is the introduction of analytical methods. The capacity to assess structural behaviour systematically, and the advancement of computer-based methods and tools, have fundamentally shaped this discipline. Structural mechanics is now a highly demanding subject, spanning both the analytical evaluation of structural behaviour and its practical application in design. Though intimately related, these two realms retain distinct focuses.

Analytically, the central question is: Given a particular structural system and specified loads, what are its resulting forces and deformations? Answering this requires a strict scientific approach – one that can, at times, give the impression that the analysis itself is the end goal. Indeed, modern computing methods and software have made these calculations routine.

On the other hand, practical design emphasises the art of applying this understanding to create efficient load-carrying solutions that are economical, functional and visually pleasing. Given a particular set of service requirements and environmental conditions, what structural concept – using appropriate materials – will best satisfy the design criteria? This is where engineering insight and creativity matter most.

Although engineering curricula tend to focus on the scientific side, aspiring structural engineers often discover too late that true mastery requires more than rote reliance on computer analysis or prescriptive codes. Equally important is the ability to ‘see’ how forces flow through a structure – to perceive its behaviour as a coherent system. Without this intuitive understanding, engineers may find themselves ill-equipped to engage meaningfully with architects and builders on real-world projects.

This two-volume book aims to bridge that gap. It explores the behaviour of a wide range of structural systems – beams, frames, arches, cables, grillages, slabs, shells, thin-walled sections and multi-storey structures – placing particular emphasis on the underlying mechanisms that allow them to support loads. It discusses traditional materials like steel and concrete alongside composite and prestressed solutions, and introduces the principles of plastic analysis, second-order theory and structural stability in a simplified manner.

Special chapters address the design of statically determinate and indeterminate plane structures, dynamic response under seismic and human-induced actions, and the treatment of shallow and deep foundations – recognising that structural design is never complete without an understanding of soil–structure interaction.

The book adopts a progressive, concept-building approach: each chapter builds on earlier material, ensuring readers establish a strong intuitive and analytical base before moving on to more advanced topics. Some background knowledge of elementary mechanics is assumed.

As Vitruvius wrote more than two millennia ago, successful structural design must satisfy four core criteria: safety, functionality, economy and beauty. Technical safety requires that a structure's capacity exceed the demand; functionality requires limiting displacements and vibrations; economy requires choosing efficient structural forms and construction methods; and beauty requires sensitivity to proportion and elegance. Achieving these ideals depends as much on an engineer's creativity and judgement as on their technical prowess.

Ultimately, this book is for anyone – student, practising engineer or architect – who wishes to gain deeper structural insight. Our hope is that it will not only enrich readers' appreciation of structural behaviour, but also help them to design with greater understanding and confidence.

Finally, we wish to thank Emerald Publishing and its editorial team, led by Dr Michael Fenton, for their invaluable guidance and support in bringing this book to fruition.

Leonidas Stavridis

Konstantinos Georgiadis

About the authors

Professor Leonidas Stavridis obtained his Diploma in Civil Engineering and his PhD from the National Technical University of Athens (NTUA). Subsequently, he attended a postgraduate course in Bridge Engineering and Prestressed Structures at the Federal Institute of Technology of Zurich (ETH) where he obtained his diploma in 1989. In 2011, after more than 20 years of teaching structural analysis, bridge engineering and structural behaviour at both undergraduate and postgraduate level, he was elected Professor of Structural Engineering and Design at NTUA.

He is the author of *Structural Systems: Behaviour and Design* published in 2010 by ICE Publishing. In addition, his publications in peer-reviewed international journals cover a wide range of topics related to static and dynamic analyses of orthotropic slabs and shallow shells, prestressed cable structures, structural behaviour of multi-storey buildings, dynamic behaviour of curved thin-walled beams, soil–structure interaction problems including prestressed foundations, the treatment of external partial prestressing in slab bridges and the behaviour of suspension and stress ribbon.

Professor Stavridis has been an active freelance structural engineering consultant for more than 45 years and has been involved in various design projects, mostly in Greece, including multi-storey buildings, prestressed concrete bridges, space covering roofs and special foundations as well as structural restoration and strengthening of traditional monumental structures.

Dr Konstantinos Georgiadis is a Chartered Engineer (CEng MICE) with over 10 years of experience in bridge engineering and structural design. He graduated from the National Technical University of Athens (NTUA) in 2011 and earned his MSc in Steel Design and Business Management with distinction from Imperial College London in 2012.

He began his professional career in 2013, gaining experience in the UK railway sector with Bouygues. In 2014, he joined AECOM as a Bridge Engineer in the structures team. In 2016, he was awarded a prestigious EPSRC scholarship to pursue a PhD in Bridge Design at Imperial College London. During his doctoral studies, he served as a Teaching Assistant in Steel Design at Imperial and worked as a part-time external consultant for AECOM. After completing his PhD in 2020, he joined Arup in Hong Kong as a Chartered Senior Bridge Engineer and later transferred to their London office.

Dr Georgiadis has led and contributed to feasibility studies, preliminary and detailed designs, structural assessments and inspections of numerous bridges, including long-span structures such as cable-stayed and suspension bridges. His international project experience spans the UK, Hong Kong, Macau, the Philippines, Switzerland, Portugal and Greece. He is well-versed in multiple design codes, including the Eurocodes, British Standards and AASHTO.

With a strong technical background, Dr Georgiadis specialises in advanced linear and nonlinear, static and dynamic analysis and is a proficient user of various finite element analysis software. He also brings substantial site experience, having supervised bridge construction, inspections and surveys. He has authored multiple technical and research papers, which have been published and presented in leading journals and international conferences and has served as member of the Scientific Committee of IABSE.

Introduction

This book aims to cultivate a deep understanding of the structural behaviour and design principles of civil engineering structures. Its focus is on fundamental concepts concerning the design and analysis of various structural types, including different types of bridges, long-span roof structures and multi-storey buildings, among others. As the selection of the appropriate construction material plays an important role in the design, this book focuses on the main construction materials – that is, steel and concrete – as well as their combinations in the form of prestressed concrete and composite structures.

It covers the conceptual design of different structural forms, fostering a solid mindset to develop a sound preliminary design. Readers will gain an understanding of load-transferring paths and the ability to estimate the internal forces in structures – both qualitatively and quantitatively – without the need to rely on computer software or design codes. Of course, these tools will eventually be used when producing the detailed design and final construction drawings, which remain the ultimate goal of any structural project.

The first volume, after an introductory overview of the structural properties of the main construction materials, addresses the equilibrium and deformability of plane skeletal structures. Chapter 2 explores statically determinate systems, while Chapter 3 focuses on statically indeterminate systems, providing readers with core concepts and tools from classical structural mechanics.

The remaining six chapters of the first volume cover the static behaviour and performance of typical plane structural systems – including simply supported and continuous beams, plane frames, arches and cable-supported structures – encouraging readers to make creative, informed decisions during their preliminary design.

Particular emphasis is given to important topics such as the use of prestressing and the practical application of the principles of plastic analysis and design, explained through the statical theorem of plasticity theory. The issue of elastic stability and of second-order effects are also explored, allowing readers to approximate these effects using equivalent first-order methods.

This book, by approaching the preliminary design process without an immediate reliance on a computer, equips readers with the ability to critically check computer-generated results during the final design process. However ultimately, the most important outcome of the whole design process should be the production of final structural drawings that fully communicate the structural configuration and the method of construction to be used.

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Chapter 1

Introductory concepts

1.1. Loads on structures

Loads constitute the raison d'être of structural systems and therefore their examination precedes anything else. Structures are formed and designed in such a way to carry safely and in a functionally satisfactory way certain loads. It lies within the responsibility of the engineer to prescribe the loads that a structure is expected to be exposed to during its design life.

The determination of loads is a difficult task, as most of the loads that act on a structure (e.g. traffic, wind, seismic etc.) vary during its design life and cannot be accurately predicted. However, there is an obvious need to design structures to undertake loads agreed in advance. For this reason, regulations and design codes define the required loads that should be carried by a structure, considering their probabilistic nature. These regulations and design codes differ from country to country (as do the loads); thus, for example, the same bridge may be designed for different loads in the UK than in the USA or Japan.

Before examining the source and nature of loads, a distinction between *static* and *dynamic loads* should be made. A load, P, is considered to act statically on a structure, when the time, t_1, needed for its full development is substantially longer than the *fundamental period, T*, of the structure. The fundamental period corresponds practically to the time it takes for a structure to perform a complete oscillation when an arbitrary deflection from its equilibrium position is imposed and then is left free to oscillate.

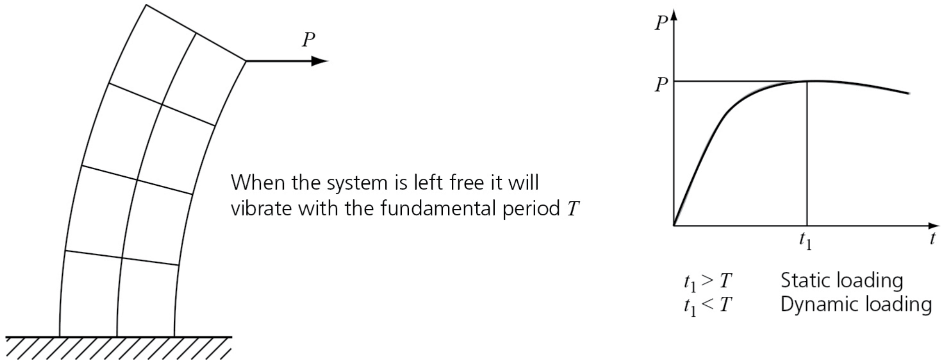
Thus, a wind gust growing from zero to its maximum value in 3 s represents a static force for a short, stiff building having a fundamental period of 0.5 s, whereas for a tall, flexible building having a fundamental period equal to 6 s, it must be considered as a dynamic loading (see Figure 1.1). The way dynamic loading is handled differs radically from that of static loading for the simple reason that, because of the induced motion, inertia forces are developed, which depend – at each instant of time – on the corresponding displacement of the structure. Due to the complexity of solving the equations of motion at every instance of time, whenever possible, it is preferred to replace dynamic loading with the equivalent static loading. This approach is generally used in design codes and regulations, where a static load is multiplied by an *impact factor* or *dynamic amplification factor* to account for its dynamic effects.

Apart from their static or dynamic character, loads can also be categorised considering their natural origin as follows.

Loads from gravity

Loads due to the weight of permanent structural components are characterised as *dead loads*, whereas loads due to the weight of permanent non-structural components (e.g. surfacing,

Figure 1.1 Load characterisation as static or dynamic



cladding etc.) are characterised as *superimposed dead loads*. Both dead and superimposed dead loads are determined according to the specific weight of materials (e.g. 25 kN/m^3 for reinforced concrete and 78.5 kN/m^3 for structural steel) and the acceleration of gravity, g, being equal to 9.81 m/sec^2. These loads are clearly considered as static.

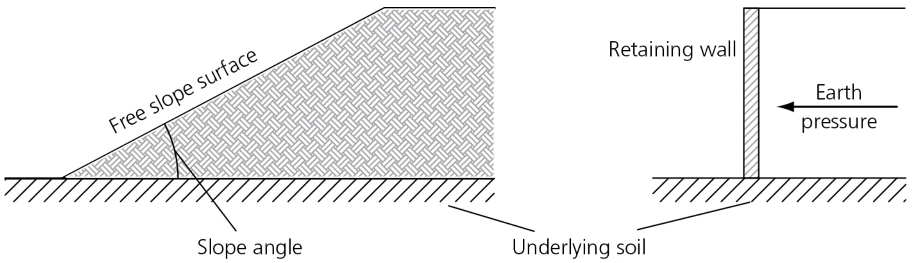
Loads due to the occupancy or use of a structure are characterised as *live loads*. This use may come from human activities (people walking on pedestrian bridges or standing in stadiums etc.), it may come from equipment installed in industrial spaces, or it may represent traffic loads due to moving vehicles on road bridges, as well as trains on railway bridges. These loads are generally calculated as the weight of the corresponding load (e.g. pedestrians, machines, vehicles, trains) and expressed either as *surface loads* or *concentrated loads*.

These loads are called ‘live’ because they may change their position on the structure. Obviously, their most adverse position for the structure should be considered in the design. Despite their dynamic nature, for simplicity live loads are usually considered to act in a static manner, using the dynamic amplification factors mentioned above. However, when the slenderness of a structure increases, the use of dynamic amplification factors is no longer satisfactory and a proper dynamic analysis should be performed.

Loads from soil

Vertical (or inclined) surfaces of a structure that come into contact with the surrounding soil are subjected to earth pressure. Earth pressure arises from the fact that a soil volume resting freely on a horizontal level forms a slope with which its free surfaces are in a state of equilibrium (see Figure 1.2). The *physical angle of slope* generally depends on the nature of the soil (e.g. sand, clay etc.), ranging from around 30° to 45°. Thus, if this free sloped surface is bounded to be retained at a different angle from the physical one – usually vertically – with a structural surface (e.g. a retaining wall), then the soil exerts pressure on this structural surface. Of course, this earth pressure increases with the divergence of the structural surface from the free slope. The earth pressure is determined as the product of the height of soil mass, times its density, multiplied by the co-called *soil factor, K*, which depends on the physical angle of slope and is generally < 1 (Stavridis and Georgiadis, 2025, Chapter 8). This situation is, in fact, similar to that of the hydrostatic pressure, where the corresponding physical angle of slope of water is obviously zero (and

Figure 1.2 Free slope surface and earth pressure



the corresponding $K = 1$). This observation explains why the earth pressure is always less than the hydrostatic pressure.

Soil supporting a structure may also exhibit settlement. This settlement might be caused by reasons that are either irrelevant to the structure (like groundwater lowering) or related to the deformability of the soil under the loading of the structure. Whatever the reason for soil settlement, it generally (not always, as will be seen in the next two chapters) constitutes a loading for structures, which is equivalent to an imposed deformation.

Finally, during an earthquake the supporting soil of a structure exhibits vibrations that are subsequently imposed on the foundations of the structure. As a result of this dynamic loading, the whole structure is subjected to movement and severe internal forces. As examined in Stavridis and Georgiadis (2025, Chapter 7), it is the imposed accelerations on the foundation that cause the internal forces in the structure, whereas the potential soil settlements during an earthquake may bring an additional load, as previously explained.

Loads from aquatic environments

Structures in an aquatic environment (e.g. marine structures, dams etc.) are generally subjected to hydrostatic pressure. Hydrostatic pressure can also act on structures placed on the ground (e.g. abutments, retaining walls) as water can also be contained in the soil.

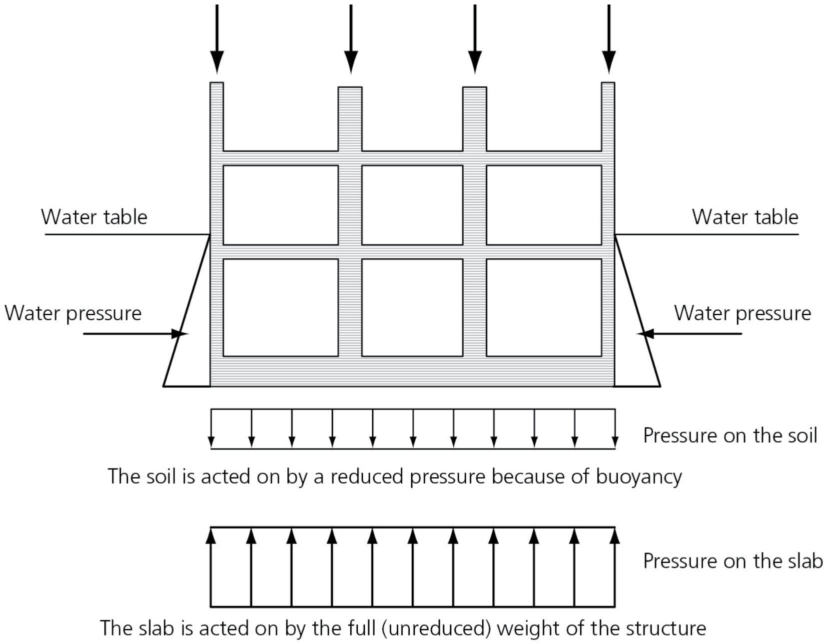
In the case of water motion relative to the structure (e.g. bridge piers in a river), additional hydrodynamic pressures come into action. This water motion can also cause scour to the soil that supports the structure and requires special attention.

Furthermore, if a structure is confined with its lower part in a soil containing groundwater (e.g. underground levels of a building), buoyancy forces are developed. These forces are nothing more than the upward hydrostatic pressure on the lower horizontal surface of the structure (see Figure 1.3). Although the buoyant force decreases the weight of the structure acting on the soil, the structure's lower surface is under the action of the total (unreduced) weight applied upwards as water pressure.

Loads from climate

Wind, snow and temperature variation are examples of loads arising from the climate.

Figure 1.3 Loads in an aquatic environment



Wind forces mainly represent a horizontal loading having a rather dynamic character. Wind forces are related to the form of a structure and its height above the earth surface. The thorough determination of wind forces in slender structures is particularly difficult as they may interact with the structure deformations, as, for example, in the case of the cable-suspended bridges, leading to complex aerodynamic calculations. However, in most ordinary simple structures, wind loads can be determined in a simplified (equivalent) static manner, as discussed above.

Snow, which is, in fact, a gravity load, is a surface load, and can be determined by considering the region as well as the altitude of the structure.

Finally, temperature variation appears either as a uniform temperature change (increase or reduction) or as a distributed change across the depth of a structural element. Non-uniform (distributed) temperature change always appears in structural elements exposed to the atmosphere (roofs, bridges etc.). As in the case of soil settlements, it must be noted that temperature variation does not always constitute a load (i.e. does not always cause internal forces) for structures, as discussed in the next two chapters.

Loads from special impact

Impact loads are generally caused by accidental actions such as a collision of a body on a structure (e.g. ship collision on a bridge pier), an explosion, fire and so on. Collision loads, despite their dynamic nature, are generally treated in design codes as equivalent static loads with the use of impact (dynamic amplification) factors, as mentioned previously. However, explosions cannot be treated in a similar manner and they require special analysis.

1.2. Main construction materials

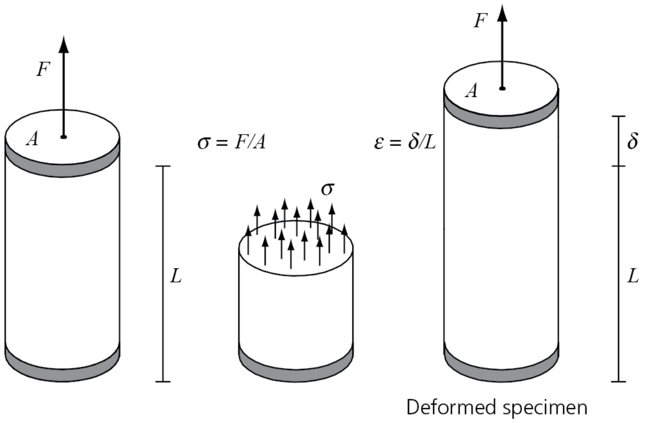
Having obtained an understanding of the loads that a structure can be exposed to, this section explains the behaviour of the main materials a structure can be made of. The *structural strength* of every material is determined on the basis of the relationship between *stress,* σ, and *strain,* ε, of an appropriate specimen subjected to pure axial force (tension or compression).

Consider a specimen with a length, L, and a cross-section, A, which is subjected to a force, F, acting along its longitudinal axis. Assuming a uniform distribution of the force on the cross-section, the concept of stress, σ, is introduced as:

$$\sigma = F/A$$

Thus, the stress, σ, is uniformly distributed over the whole cross-section, A, of the specimen (see Figure 1.4). The force, F, concerns the whole specimen externally, whereas the stress, σ, characterises the internal state of its longitudinal fibres. As a force is generally expressed in kilonewtons (kN) and an area in square metres (m^2), the unit of stress is kN/m^2. However, it is also common to use newtons and square millimetres (N/mm^2) to express stress, which is equal to megapascals (MPa).

Figure 1.4 The concept of stress and strain



The application of the force, F, results in an elongation or shortening, δ, of the specimen, depending on whether the force is tensile or compressive. Thus, the concept of strain, ε, is introduced as:

$$\varepsilon = \delta/L \text{ (non-dimensional number)}$$

The relationship between stress (σ) and strain (ε) can be obtained experimentally by loading an appropriate specimen progressively and at a constant rate, starting from a null loading level ($F = 0$) and continuously recording the values of stress (σ) and strain (ε) in a diagram of two orthogonal axes (as they are calculated on the basis of the measured values of force, F, and displacement, δ). It should be noted that in reality the experiment is usually carried out by imposing on the specimen a specific strain, ε, rather than a stress, σ. The rate of application of ε ($d\varepsilon/dt$) is about equal to $5 \cdot 10^{-5}\ s^{-1}$.

1.2.1 Steel

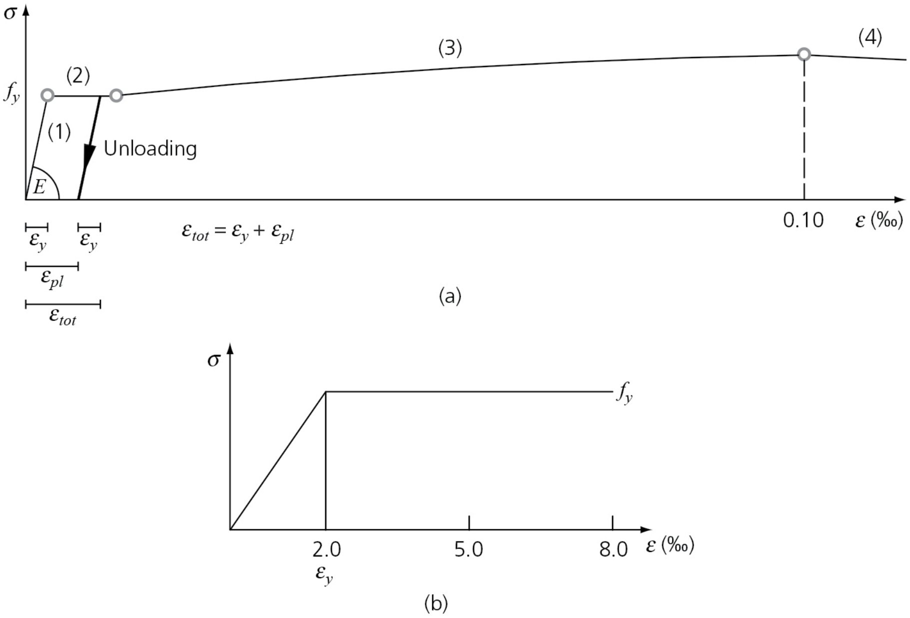
Steel is one of the most common construction materials (used for steel structures, concrete reinforcement etc.). The experimentally obtained relationship between its stress (σ) and strain (ε) is shown in Figure 1.5(a). The diagram represents a test done in tension but applies equally for compression. For each pair of values (σ, ε) on the diagram, the force, F, and the corresponding deformation, δ, can be determined as:

$$F = \sigma \cdot A \tag{a}$$

and

$$\delta = \varepsilon \cdot L \tag{b}$$

Figure 1.5 (a) Stress–strain diagram of steel in tension; (b) idealised stress–strain diagram for steel



Note: f_y, yield stress or yield strength; ε_{pl}, plastic strain; ε_{tot}, total strain; ε_y, yield strain.

Four distinct regions are observed in the stress–strain (σ–ε) curve (see Figure 1.5(a)). In region (1), the stress, σ, is proportional to the strain, ε, which means (as will be shown later) that the corresponding force, F, is also proportional to the deformation, δ. This fundamental relationship is known as *Hooke's law*, expressed as:

$$\sigma = E \cdot \varepsilon \tag{c}$$

The parameter, E, is called the *modulus of elasticity*, derived graphically as the slope of the curve in region (1). The modulus of elasticity, E, represents the most fundamental structural characteristic of a material. It can be thought of as a measure of the resistance offered by it against its unit elongation or shortening. In the case of steel, the value of the modulus of elasticity, E_s, is about 210 000 MPa.

Substituting the above Equations (a) and (b) in (c), the relationship between the external force, F, and deformation, δ, of the specimen can be obtained as:

$$F = (EA/L) \cdot \delta$$

This relation shows that the corresponding force, F, is proportional to the deformation, δ, as mentioned above. The value (EA/L) is called the *axial stiffness* of the element and is commonly expressed in kN/m. Thus, the axial stiffness defines the force in kN required to deform (either elongate or shorten) a particular specimen by 1 m – in other words, the resistance (stiffness) in kN that is offered by the specimen if it is forced to an axial deformation of 1 m. The above relationship can also be written as:

$$\delta = F \cdot (L/EA)$$

which shows how the axial deformation, δ, of the specimen can be calculated from the externally applied force, F. This equation is known also as Hooke's law. The expression (L/EA) is called *axial flexibility* or *deformability* (m/kN). It gives the axial deformation of the specimen in m, if an axial force of 1 kN is applied. It is clear that these two concepts (and values) of stiffness and flexibility are inversely related (see Section 2.3.8).

The behaviour of the material in region (1) is called *linear behaviour* and is governed by an absolute proportionality between force, F (or stress σ), and deformation, δ (or strain, ε). This means that if the force, F (or stress, σ), is multiplied by a factor, then the deformation, δ (or strain, ε), is also applied by the same factor. However, this linear behaviour holds only up to a certain level of stress, f_y, at which point region (2) of the behaviour of steel begins (see Figure 1.5(a)). At this level of stress, the corresponding strain is ε_y.

For steel used in structures (structural steel), the value of f_y typically ranges from 235 to 355 MPa, whereas for steel bars used as concrete reinforcement (reinforcing steel), the value of f_y typically ranges from 420 to 500 MPa. The corresponding ε_y – that is, (f_y/E_s) – is in the order of 2‰.

Once the stress has reached the value of f_y, the specimen can be further deformed without showing any increase of the applied force (and stress). This point defines the beginning of region (2). In this region, the material exhibits zero stiffness (the slope of the diagram becomes zero), which means it 'yields'. For this reason, the stress, f_y, is called *yield stress* or *yield strength*.

After this state, the material acquires some stiffness again, as can be seen from the small slope with which region (3) begins. A further increase in strain, ε, causes a small increase in stress, σ (i.e. the force, F); this behaviour of the steel is called *steel hardening*. However, this minimal stiffness in practice can be considered zero (as in region (2)) and, consequently, the very small increase in stress, f_y, in this region can be simply ignored.

Up to the point where the stress reaches the value of f_y (i.e. region (1)), if the load is removed from the specimen, the deformation, δ (as well as the strain, ε) will return to zero. In other words, up to this point the deformation is reversible; for this reason, region (1) is called the *elastic region*.

After region (1), the elastic behaviour is no longer valid. Unloading of the specimen in region (2) or (3) will lead to a course parallel to the elastic region and as a result to a *remaining deformation* represented by the strain, ε_{pl} (see Figure 1.5(a)). Thus, after yielding, the specimen cannot return to its initial length and will be either a bit longer or a bit shorter, depending on whether the applied stress was tensile or compressive. Therefore, it can be said that the material has undergone a *plastic deformation* – that is, *plastic strain,* ε_{pl}. The total deformation of the specimen, before unloading, is represented by the total strain, ε_{tot} (see Figure 1.5(a)), which is:

$$\varepsilon_{tot} = \varepsilon_y + \varepsilon_{pl}$$

The ability of steel to deform without breaking under the constant yield stress, f_y, is a very important structural property called *ductility*. This property holds not only for steel structures, but also for reinforced or prestressed concrete structures. It is very beneficial for the load-carrying mechanism of structures especially for seismic loads, as will be explained in the following chapters.

The state of yield can be considered to extend up to values of strain, ε, of about 10% where region (4) begins. In region (4), the increase of deformation, δ (i.e. strain, ε), requires increasingly less force, F (i.e. stress, σ), until the point where the specimen fails. This region does not have any practical interest for the design of structures.

From all the above, it is concluded that the maximum level of stress for steel cannot practically exceed the value f_y. Thus, in practice, steel can be considered as a homogeneous linear elastic-fully plastic material. The idealised diagram (σ–ε) that describes this behaviour is shown in Figure 1.5(b). For practical design purposes, the maximum value of strain, ε, is taken as 8‰ (see Figure 1.6(a)). This idealised material behaviour is generally used in practice.

The stress–strain diagram described so far refers to naturally hardened steel, the strength of which is entirely due to its chemical composition. This quality of steel is generally used for both structural elements and concrete reinforcement (see Figure 1.6(a)). However, when a steel of very high strength is needed (e.g. in the use of tendons in prestressed concrete, as explained in Section 1.4), this is obtained through a cold forming procedure.

The stress–strain diagram for high-strength steel is shown in Figure 1.6(c). Comparing this with the graph for normal steel, it is immediately observed that there is no yield plateau, which means that there is no clearly defined yield stress in the way that has been previously established. In this case, the yield stress, f_{Py}, is defined as the stress that corresponds to the point where the removal of the load from the specimen will lead to a remaining plastic strain, $\varepsilon_{pl} = 2‰$. This yield stress can be considered constant after a value of $\varepsilon_{pl} \approx 8‰$ has been reached, as shown in the idealised diagram in Figure 1.6(b). Thus, for practical purposes, the actual ultimate yield strength of prestressing steel beyond f_{Py} can be ignored.

1.2.2 Concrete

Concrete is also one of the most common construction materials and its structural behaviour is examined here. Concrete mainly consists of cement, sand, gravel and water and, like steel, is generally considered a homogeneous material. However, in contrast with steel (which exhibits practically the same stress–strain diagram for both tension and compression), the behaviour of concrete under tension is totally different from that in compression.