# ENERGY POWER RISK

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# ENERGY POWER RISK: DERIVATIVES, COMPUTATION AND OPTIMIZATION

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*To Kathy, Matthew, Claire and Rachel*

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# Contents

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# List of Figures

**Chapter 8**

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# List of Tables

# Notations

The notation used is as follows:

| | |
|---|---|
| $GBM$ | Geometric Brownian motion |
| $BM$ | Brownian motion |
| $W_t$ | Brownian motion at time $t$ – the term may be nonzero |
| $\rho$ | The correlation coefficient |
| $E[x]$ | The expectation value of $X$ |
| $Var[X]$ | The variance of $X$ |
| $Cov[X,Y]$ | The covariance between $X$ and $Y$ |
| $Cov[X]$ | The covariance between the variates contained in the vector $X$ |
| $\sigma$ | The volatility. Since assets are assumed to follow GBM, it is computed as the *annualized* standard deviation of the $n$ continuously compounded returns |
| $N_1(a)$ | The univariate cumulative normal distribution function. It gives the cumulative probability, in a standardized univariate normal distribution, that the variable $x_1$ satisfied $x_1 \leq a$ |
| $N_2(a,b,\rho)$ | The bivariate cumulative normal distribution. It gives the cumulative probability, in a standardized bivariate normal distribution, that the variables $x_1$ and $x_2$ satisfy $x_1 \leq a$ and $x_2 \leq b$ when with correlation coefficient between $x_1$ and $x_2$ is $\rho$ |
| $r$ | The risk free interest rate |
| $q$ | The continuously compounded dividend yield |
| $S_{it}$ | The $i$th asset price at time $t$ |
| $I_{nn}$ | The $n$ by $n$ unit matrix |
| $\Lambda(\mu,\sigma^2)$ | A lognormal distribution with parameters $\mu$ and $\sigma^2$. If $y = \log(x)$ and $y \sim N\left(\mu,\sigma^2\right)$, then the distribution for $x = e^y$ is $x \sim \Lambda(\mu,\sigma^2)$. We have $E[x] = \exp\left(\mu + (\sigma^2/2)\right)$ and $Var[x] = \exp\left(2\mu + \sigma^2\right)\left(\exp\left(\sigma^2\right) - 1\right)$ |
| $\log(x)$ | The natural logarithm of $x$ |
| $N(a,b)$ | Normal distribution, with mean $a$ and variance $b$ |
| $dW_t$ | A normal variate (sampled at time $t$) from the distribution $N(0,dt)$, where $dt$ specified time interval e.g., $dx = \mu dt + dW_t$ |
| $dZ_t$ | A normal variate (sampled at time $t$) from the distribution $N(0,1)$. Note: The variate $d\psi = \sqrt{dt}\, dZ_t$ has the same distribution as $dW_t$ |

| | |
|---|---|
| $IID$ | Independently and identically distributed |
| $\mathcal{U}(a, b)$ | The uniform distribution, with lower limit $a$ and upper limit $b$ |
| $\lvert x \rvert$ | The absolute value of the variable $x$ |
| $PDF$ | The probability density function of a given distribution |
| $x \wedge y$ | The minimum of $x$ and $y$, that is, $\min(x, y)$ |

# Preface

The aim of this book is to provide readers with sufficient knowledge to understand and create quantitative models for energy/power risk and derivative valuations. The topics covered include the mathematics of stochastic processes, assets optimization, Markowitz portfolio optimization, derivative valuation, and financial engineering using C++. One could write a separate book on each of these subjects, and therefore of necessity their coverage in this book could be considered to be an introduction. However, I trust that readers will find the book a useful reference and building block for their projects.

I would like to thank my wife Kathy for her support, and also my daughter Rachel for her expert help in creating some of the figures.

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Chapter 1

# Overview

This book covers energy risk, derivative valuation, and also software development. It aims to be practical and contains many Code Excerpts, in both C++ and Microsoft Excel VBA, that can readily be incorporated into software projects. The reader is assumed to have a basic understanding of both linear algebra and calculus; a comprehensive appendix is also included for ease of reference. There is first an introduction to the mathematics of Brownian motion and stochastic processes. All the equations and results are derived from first principles, and a large selection of problems (with answers in the appendix) have been included to test the reader's understanding. The topics dealt with are used later on in the book and include the following: Brownian motion, Geometric Brownian motion, Ito's lemma, Ito's Isometry, Ito's product and quotient rules, Ito's lemma for multi-asset geometric Brownian motion, the Ornstein Uhlenbeck process, and the Brownian Bridge.

The book then deals with the mathematics of spot and forward curve commodity models, Merton's jump diffusion model, non-normal distributions, and pricing multi-asset European and American derivatives.

A chapter on Markowitz portfolio optimization has been included since, although this method is widely used in finance for stocks and shares, it also has applications in energy risk. Examples are provided using a numerical optimization component that the author has developed; it allows the Objective Function and Constraints Function to be written in Microsoft Excel VBA. Free Excel VBA optimization demonstration software is included with the book.

There is also a section on software engineering which illustrates how to create C++ vector and random number classes that facilitate the development of energy risk and derivative pricing software. Examples of using the classes are provided and Code Excerpts are supplied so that readers can adapt these to suit their own requirements.

The book also contains details of the author's recent research on UK power contracts. This was published in a series of *Energy Risk Magazine* articles, and cover the topics of power imbalance, renewable generation (wind and solar), intraday storage, and demand optionality.

In recent years, there has been a marked increase in the amount of renewable generation delivered to the UK power market. Figures from the National Grid, see (Elexon (2018)), show that in January 2015, wind power provided 14% of Britain's energy, and between 5 and 11 January, wind power supplied 31% of Britain's energy requirements. In addition, there has also been a large increase in the installed national solar Photovolataic (PV) capacity with substantial PV generation during peak hours in the summer month; on June 30, 2015, the peak half hour generation was about 7 GW, see (Elexon (2018)). More recent data show that in 2017, between July and September, 54.4% the UK's electricity came from nuclear power stations and renewables. This was a result of the rapid growth in solar and wind power. During the same period in 2016, the share for low carbon electricity stood at 50%, and in 2015 it was 45%. Furthermore, in April 2018, the United Kingdom was powered without coal for three days in a row; the last time this happened was in the nineteenth century.

However, in contrast to other forms of renewable energy, such as hydroelectricity, tidal power, and biomass, wind and solar energy present additional problems because of their intermittency and the lack of affordable storage technology. The increase in renewable generation has caused higher power price volatility. This is because when there is of plenty of wind and solar generation, the half hour price will be low, but on days of low or intermittent renewable generation, the power price can exhibit very high price spikes. The current average half hourly wholesale price is around £40/MWh, but there have been occasions when the price has either become negative or spiked above £1,000/MWh. However, one cannot assume that future prices will be similar to those of the past. This is because the UK generation mix will change and the system operator will create new products to incentivize generators and consumers to behave in what it considers to be a beneficial manner. In Chapter 3, we capture these effects by using a Monte Carlo–based fundamental power stack model of the United Kingdom that includes national generation and demand forecasts.

Since cash flow is the product of volume (generation/consumption) and price, the benefit or risk associated with an energy contract depends on these parameters. For example, the imbalance risk associated with a wind power contract is determined by the product of the uncertainty in wind generation and the system price, see Chapter 7.

Half hourly power price volatility also provides the opportunity to use generation, demand side response (DSR), and storage to create value. This involves making optimal decisions to increase power consumption (store or stop generating) when the half hour price is low, and decrease consumption (release or start generating) when the half power price is high. These decisions are treated as multiple exercise American options, and they are valued using the Longstaff Schwartz regression approach. It should be mentioned that the valuation of DSR and storage for a given site is complex and depends on the interplay between factors such as: the storage and generation response times, whether the site has wind or solar PV generation, and the fraction of the renewable power that the site can consume.

Chapter 2

# Brownian Motion and Stochastic Processes

## 2.1. Brownian Motion

Many of the fundamental properties of Brownian motion were discovered by Paul Levy (Levy, 1939), and (Levy, 1948) and the first mathematically rigorous treatment was provided by Norbert Wiener (Wiener, 1923) and (Wiener, 1924). Karatzas and Shreve (2000) is an excellent text book on the theoretical properties of Brownian motion.

Brownian motion is also called a *random walk*, a Wiener process, or sometimes (more poetically) the *drunkards walk*. We will now present the three fundamental properties of Brownian motion.

### *2.1.1. The Properties of Brownian Motion*

In formal terms, a process is $W = (W_t : t \geq 0)$ is (one-dimensional) Brownian motion if

(1) $W_t$ is continuous, and $W_0 = 0$,
(2) $W_t \sim N(0, t)$,
(3) The increment $dW_t = W_{t+dt} - W_t$ is normally distributed as $dW_t \sim N(0, dt)$, so $E[dW_t] = 0$ and $Var[dW_t] = dt$. The increment $dW_{dt}$ is also independent of the history of the process up to time $t$.

From (iii), we can further state that, since the increments $dW_t$ are independent of past values $W_t$, a Brownian process is also a *Markov* process. In addition, we shall now show that Brownian process is also a martingale process.

In a martingale process $P_t, t \geq 0$, the conditional expectation $E[P_{t+dt}|\mathcal{F}_t] = P_t$, where $\mathcal{F}_t$ is called the *filtration* generated by the process and contains the information learned by observing the process up to time $t$. Since for Brownian motion we have

$$\begin{aligned} E[W_{t+dt}|\mathcal{F}_t] &= E[(W_{t+dt} - W_t) + W_t|\mathcal{F}_t] = E[W_{t+dt} - W_t] + W_t \\ &= E[dW_t] + W_t = W_t, \end{aligned}$$

where we have used the fact that $E[dW_t] = 0$. Since $E[W_{t+dt}|\mathcal{F}_t] = W_t$, the Brownian motion $Z$ is a martingale process.

Using property (iii), we can also derive an expression for the covariance of Brownian motion. The independent increment requirement means that for the $n$ times

$0 \le t_0 < t_1 < t_2, \ldots, t_n < \infty$ the random variables $W_{t_1} - W_{t_0}, W_{t_2} - W_{t_1}, \ldots, W_{t_n} - W_{t_{n-1}}$ are independent. So

$$Cov\left[W_{t_i} - W_{t_{i-1}}, W_{t_j} - W_{t_{j-1}}\right] = 0, \quad i \neq j \tag{2.1.1}$$

We will show that $Cov\,[W_s, W_t] = s \wedge t$

*Proof*:

Using $W_{t_0} = 0$, and assuming $t \ge t$ we have

$$Cov\left[W_s - W_{t_0}, W_t - W_{t_0}\right] = Cov\,[W_s, W_t] = Cov\,[W_s, W_s + (W_t - W_s)]$$

From Appendix B.3.2 we have

$$Cov\,[W_s, W_s + (W_t - W_s)] = Cov\,[W_s, W_s] + Cov\,[W_s, W_t - W_s]$$

$$= Var\,[W_s] + Cov\,[W_s, W_t]$$

$$= s + Cov\,[W_s, W_t - W_s]$$

Now since

$$Cov\,[W_s, W_t] = Cov\left[W_s - W_{t_0}, W_t - W_s\right] = 0$$

where we have used Eq. (2.1.1) with $n = 2, t_1 = t_s$ and $t_2 = t$.

We thus obtain

$$Cov\,[W_s, W_t] = s$$

So

$$Cov\,[W_s, W_t] = s \wedge t \tag{2.1.2}$$

We will now consider the Brownian increments over the time interval $dt$ in more detail. Let us first define the process $X$ such that

$$dX_t = dW_t, \tag{2.1.3}$$

where $dW_t$ is a random variable drawn from a normal distribution with mean zero and variance $dt$, which we denote as $dW_t \sim N(0, dt)$. Eq. (2.1.3) can also be written in the equivalent form:

$$dX_t = \sqrt{dt}\; dZ, \tag{2.1.4}$$

where $dZ$ is a random variable drawn from a *standard* normal distribution (i.e., a normal distribution with zero mean and unit variance).

Eqs. (2.1.3) and (2.1.4) give the incremental change in the value of $X$ over the time interval $dt$ for *standard* Brownian motion.

We shall now generalize these equations slightly by introducing the extra (*volatility*) parameter $\sigma$ which controls the variance of the process. We now have

$$dX_t = \sigma dW_t, \tag{2.1.5}$$

where $dW_t \sim N(0, dt)$ and $dX_t \sim N(0, \sigma^2\, dt)$. Eq. (2.1.5) can also be written in the equivalent form:

$$dX_t = \sigma \sqrt{dt}\, dZ, \;\; dZ \sim N(0,1) \tag{2.1.6}$$

or equivalently

$$dX_t = \sqrt{dt}\, d\hat{Z}, \;\; d\hat{Z} \sim N(0,\sigma^2) \tag{2.1.7}$$

### *2.1.2. A Brownian Model of Asset Price Movements*

The first attempt at using Brownian motion to describe asset price movements was provided by Bachelier (1900). This however only had limited success because the *significance* of a given *absolute* change in asset price depends on the original asset price. This lead to the idea of using the relative price changes and can be formalized by defining a quantity called the *return*, $R_t$, of an asset at time $t$. The return $R_t$ is defined as follows:

$$R_t = \frac{S_{t+dt} - S_t}{S_t} = \frac{dS_t}{S_t}, \tag{2.1.8}$$

where $S_{t+dt}$ is the value of the asset at time $t + dt$, $S_t$ is the value of the asset at time $t$, and $dS_t$ is the change in value of the asset over the time interval $dt$. The percentage return $R^*$, over the time interval $dt$, is simply defined as $R^* = 100 \times R_t$.

We are now in a position to construct a simple Brownian model of asset price movements.

The asset *return* at time $t$ is given by

$$R_t = \frac{dS_t}{S_t} = \mu dt + \sigma dW_t,\; dW_t \sim N(0, dt) \tag{2.1.9}$$

or equivalently

$$dS_t = S_t \mu dt + S_t \sigma dW_t. \tag{2.1.10}$$

The process given in Eqs. (2.1.9) and (2.1.10) is termed *geometric Brownian motion*; which we will abbreviate as GBM. This is because the relative (rather than absolute) price changes follow Brownian motion.

## 2.2. Ito's Formula (or Lemma)

In this section, we will derive Ito's formula; a more rigorous treatment can be found in (Karatzas and Shreve (2000)).

Let us consider the stochastic process $X$:

$$dX = adt + bdW = adt + b\sqrt{dt}\, dZ,\; dZ \sim N(0,1),\; dW \sim N(0, dt) \tag{2.2.1}$$

where $a$ and $b$ are constants. We want to find the process followed by a function of the stochastic variable X, that is, $\phi(X,t)$. This can be done by applying a Taylor expansion, up to second order, in the two variables $X$ and $t$ as follows:

$$\phi^* = \phi + \frac{\partial \phi}{\partial t}dt + \frac{\partial \phi}{\partial X}dX + \frac{1}{2}\frac{\partial^2 \phi}{\partial X^2}dX^2 + \frac{1}{2}\frac{\partial^2 \phi}{\partial t^2}dt^2 + \frac{\partial \phi}{\partial X \partial t}dXdt \qquad (2.2.2)$$

where $\phi^*$ is used to denote the value $\phi(X+dX, t+dt)$, and $\phi$ denotes the value $\phi(X,t)$. We will now consider the magnitude of the terms $dX^2$, $dXdt$, and $dt^2$ as $dt \to 0$. First

$$dX^2 = (adt + b\sqrt{dt}\ dZ)(adt + b\sqrt{dt}\ dZ) = a^2dt^2 + 2ab\ dt^{3/2}\ dZ + b^2\ dt\ dZ^2$$

then

$$dXdt = adt^2 + b\ dt^{3/2}\ dZ$$

So as $dt \to 0$, and ignoring all terms in $dt$ of order greater than 1, we have

$$dX^2 \sim b^2dt\ dZ^2,\ \ dt^2 \sim 0,\ \text{and}\ dXdt \sim 0$$

Therefore Eq. (2.2.2) can be rewritten as

$$d\phi = \frac{\partial \phi}{\partial t}dt + \frac{\partial \phi}{\partial X}dX + \frac{1}{2}\frac{\partial^2 \phi}{\partial X^2}E\left[dX^2\right] \qquad (2.2.3)$$

where $d\phi = \phi^* - \phi$, and we have replaced $dX^2$ by its expected value $E\left[dX^2\right]$. Now

$$E[dX^2] = E[b^2dt\ dZ^2] = b^2dtE[dZ^2] = b^2dt,$$

where we have used the fact that, since $dZ \sim N(0,1)$, the variance of $dZ$, $E[dZ^2]$, is by definition equal to 1. Using these values in Eq. (2.2.3) and substituting for $dX$ from Eq. (2.2.1), we obtain

$$d\phi = \frac{\partial \phi}{\partial t}dt + \frac{\partial \phi}{\partial X}(adt + bdw) + \frac{b^2}{2}\frac{\partial^2 \phi}{\partial X^2}\ dt, \qquad (2.2.4)$$

This gives Ito's formula

$$d\phi = \left(\frac{\partial \phi}{\partial t} + a\frac{\partial \phi}{\partial X} + \frac{b^2}{2}\frac{\partial^2 \phi}{\partial X^2}\right)dt + \frac{\partial \phi}{\partial X}b\ dW \qquad (2.2.5)$$

In particular, if we consider the geometric Brownian process,

$$dS = \mu Sdt + \sigma SdW,$$

where $\mu$ and $\sigma$ are constants, then substituting $X = S$, $a = \mu S$, and $b = \sigma S$ into Eq. (2.2.1) yields

$$d\phi = \left(\frac{\partial \phi}{\partial t} + \mu S\frac{\partial \phi}{\partial S} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 \phi}{\partial S^2}\right)dt + \frac{\partial \phi}{\partial S}\sigma SdW. \qquad (2.2.6)$$

Eq. (2.2.6) describes the change in value of a function $\phi(S, t)$ over the time interval $dt$, when the stochastic variable $S$ follows GBM. This result has very important applications in the pricing of derivatives. Here the function $\phi(S, t)$ is taken as the price of a derivative, $f(S, t)$, that depends on the value of an underlying asset $S$, which is assumed to follow GBM. In Chapter 4, we will use Eq. (2.2.6) to derive the (Black–Scholes) partial differential equation that is satisfied by the price of a derivative.

We can also use Eq. (2.2.3) to derive the process followed by $\phi = \log(S_t)$. We have

$$\frac{\partial \phi}{\partial S_t} = \frac{\partial \log(S_t)}{\partial S} = \frac{1}{S}, \quad \frac{\partial^2 \phi}{\partial S_t^2} = \frac{\partial}{\partial S_t}\left(\frac{\partial \log(S_t)}{\partial S_t}\right) = \frac{\partial}{\partial S_t}\left(\frac{1}{S_t}\right) = -\frac{1}{S_t^2}$$

$$\frac{\partial \phi}{\partial t} = \frac{\partial \log(S_t)}{\partial t} = 0$$

So

$$d(\log(S_t)) = \nu dt + \sigma\, dW_t, \text{ where } \nu = \mu - \frac{\sigma^2}{2} \tag{2.2.7}$$

Integrating Eq. (2.2.7) yields

$$\int_{t=t_0}^{T} d(\log(S_t)) = \int_{t=t_0}^{T} \nu dt + \int_{t=t_0}^{T} \sigma\, dW_t$$

so

$$\log(S_T) - \log\left(S_{t_0}\right) = \nu T + \sigma\, W_T \tag{2.2.8}$$

where we have used $t_0 = 0$ and $W_{t_0} = 0$

We obtain

$$\log\left(\frac{S_T}{S_{t_0}}\right) \sim N\left(\nu T, \sigma^2 T\right) \tag{2.2.9}$$

and so

$$\log\left(\frac{S_T}{S_{t_0}}\right) = \nu T + \sigma\, W_T \tag{2.2.10}$$

The solution to the GBM in Eq. (2.2.7) is

$$S_T = S_{t_0} \exp(\nu T + \sigma\, W_T), \qquad \nu = \mu - \frac{\sigma^2}{2} \tag{2.2.11}$$

The asset value at time $t + dt$ can therefore be generated from its value at time $t$ by using

$$S_{t+dt} = S_t \exp\{\nu dt + \sigma\, dW_t\}$$

We have shown that if the asset price follows GBM, then the logarithm of the asset price follows standard Brownian motion. Another way of stating this is that, over the time interval $dt$, the change in the logarithm of the asset price is a Gaussian distribution with mean $\left(\mu - (\sigma^2/2)\right)\, dt$, and variance $\sigma^2 dt$.

These results can easily be generalized to include time varying drift and volatility. Now instead of Eq. (2.2.7) we have

$$dS_t = S_t \mu_t t dt + S_t \sigma_t dW_t, \tag{2.2.12}$$

which results in

$$d\left(\log\left(S_t\right)\right) = \nu_t dt + \sigma_t \, dW_t \tag{2.2.13}$$

so

$$\int_{t=t_0}^{T} d\left(\log\left(S_t\right)\right) = \int_{t=t_0}^{T} \nu_t dt + \int_{t=t_0}^{T} \sigma_t \, dW_t$$

which results in the following solution for $S_T$

$$S_T = S_{t_0} \exp\left(\int_{t=t_0}^{T} \nu_t dt + \int_{t=t_0}^{T} \sigma_t \, dW_t\right) \quad \text{where } \nu_t = \mu_t - \frac{\sigma_t^2}{2} \tag{2.2.14}$$

The results presented in Eqs. (2.2.11) and (2.2.14) are very important and will be referred to in later sections of the book.

## 2.3. Girsanov's Theorem

This theorem states that for any stochastic process $k(t)$ such that $\int_{s=0}^{t} k(s)^2 ds < \infty$ then the Radon–Nikodym derivative $d\mathbb{Q}/d\mathbb{P} = \rho(t)$ is given by

$$\rho(t) = \exp\left\{\int_{s=0}^{t} k(s) dW_s^P - \frac{1}{2}\int_{s=0}^{t} k(s)^2 ds\right\} \tag{2.3.1}$$

where $W_t^P$ is Brownian motion (possibly with drift) under probability measure $\mathbb{P}$, see (Baxter and Rennie (1996)). Under probability measure $\mathbb{Q}$ we have

$$W_t^Q = W_t^P - \int_{s=0}^{t} k(s) ds \tag{2.3.2}$$

where $W_t^Q$ is also Brownian motion (possibly with drift).

We can also write

$$dW^P = dW^Q + k(t) dt \tag{2.3.3}$$

Girsanov's theorem thus provides a mechanism for changing the drift of a Brownian motion.