ICE Core Concepts: Hydraulics for Civil Engineers

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Second edition

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Preface

Preface to the 2nd edition

The principal changes are the addition of two new chapters, covering hydraulic measurement and control structures and sediment transport, as well as extending the scope of consideration of modelling to include mathematical models. The chapter on surface drainage systems now gives greater emphasis to sustainable drainage systems (SuDS). The previous chapter on two-dimensional ideal flow has been removed.

Climate change is an increasing concern. Within Chapter 8 there is an outline of developments in understanding the effects of climate change and the allowances that are expected to be made in fluvial engineering design. The issues are also reflected in the chapters on surface water drainage and coastal hydraulics.

I would like to thank Alison Lees for her thoroughness and patience during the copy-editing process.

Preface to the 1st edition

I have entitled this volume ‘Hydraulics’ rather than ‘Fluid Mechanics’ because the latter implies the treatment of gases as well as liquids. Previous textbook authors have not always made the distinction.

My teaching experience has shown me that students are often put off hydraulics by the mathematics contained within the subject, rather than the subject itself. Rather than attempting to avoid the mathematics by saying things like ‘it is easy to show that …’, I have shown full workings wherever possible.

In a subject that is largely based on simplifying theory, it is tempting to end up with a simple design, with little regard to aesthetics or environmental impact. Hence hydraulic engineers have often produced (for example) inappropriate concrete-lined river channels of rectangular cross-section, and specific comment on this is contained in Chapter 5. The traditional hydraulic engineers’ desire for piped solutions is also challenged by moves towards the sustainable drainage systems included in Chapter 8 [Chapter 9 in the second edition].

Teachers in universities and colleges are becoming alarmed at the failure of students to give proper attribution to work they use. Not all authors of textbooks have been innocent in this regard, however. For this reason, I have attempted to identify the original publications on which much current practice is based. Hence, many of the references quoted go back to the nineteenth century. In turn, I apologise if I have not properly attributed material myself and would be grateful to learn of any such transgressions on my part.

List of symbols

		Used in chapters											
		1	2	3	4	5	6	7	8	9	10	11	12
A	area	✗	✗	✗	✗	✗	✗	✗		✗		✗	
A_{gr}	value of F_{gr} at initial motion											✗	
A_p	paved area									✗			
a	amplitude of wave										✗		
a	acceleration												✗
B	width	✗				✗	✗	✗				✗	✗
B_c	width of spillway corrected for presence dividing piers							✗					
b	expression of slope angle (1 in b)					✗							
b	width of weir notch							✗					
b_e	effective width							✗					
BFI	baseflow index								✗				
C	Chézy coefficient					✗							
C	runoff coefficient									✗			
C	coefficient in Ackers and White equation											✗	
C	suspension concentration											✗	
C_D	drag coefficient											✗	
C_d	coefficient of discharge		✗					✗		✗			✗
C_p	pressure coefficient												✗
C_R	routing coefficient									✗			
C_V	volumetric runoff coefficient									✗			
C_v	correction coefficient							✗					
C_0	deep water celerity										✗		
c	wave celerity			✗							✗		

		Used in chapters											
		1	2	3	4	5	6	7	8	9	10	11	12
CWI	catchment wetness index								✗				
D	height	✗											
D	diameter			✗		✗						✗	✗
D_*	dimensionless particle size											✗	
d	still water depth										✗		
DPR_{CWI}	dynamic runoff component								✗				
DPR_{RAIN}	dynamic runoff component								✗				
E	elastic modulus			✗									
E	energy transferred in turbine				✗								
E	specific energy					✗	✗						
E	wave energy										✗		
$\bar{E}$	specific energy of wave										✗		
e	wall thickness			✗									
Eu	Euler number												✗
F	force	✗	✗		✗								✗
F_D	frictional drag force											✗	
F_{gr}	sediment mobilisation factor											✗	
f	friction factor (North America)			✗									
FARL	measure of attenuation effect of water bodies									✗			
Fr	Froude number					✗	✗	✗				✗	✗
G_{gr}	dimensionless transport rate											✗	

		Used in chapters											
		1	2	3	4	5	6	7	8	9	10	11	12
g	acceleration due to gravity	✗	✗	✗	✗	✗	✗	✗		✗	✗	✗	✗
H	head				✗								
H	water depth					✗	✗	✗				✗	✗
H	waveheight										✗		
H_c	critical depth					✗	✗	✗					
H_d	design head							✗					
H_{max}	highest wave in a storm										✗		
H_n	average of highest n% of wave heights										✗		
H_{rms}	root mean square waveheight										✗		
H_s	significant waveheight										✗		
H_0	deep water waveheight										✗		
h	depth; height; head	✗	✗					✗		✗		✗	
h_e	effective head							✗					
h_f	head loss due to friction			✗									✗
h_G	depth to centroid	✗											
h_L	head loss at change of section			✗									
h_P	depth to centre of pressure	✗											
h_p	sudden change in bed level						✗	✗					
$HOST_n$	proportion of area occupied by soil type									✗			
I_G	second moment of area	✗											
i	rainfall intensity									✗			
K	bulk modulus of elasticity			✗									

		Used in chapters											
		1	2	3	4	5	6	7	8	9	10	11	12
K	coefficient of roughness (channel bed)					✗							
K	coefficient in nappe equation							✗					
K_C	Keulegan–Carpenter number											✗	
K_s	sluice coefficient						✗						
K_s	shoaling coefficient										✗		
$K_{1,2,3}$	correction factors in scour equation											✗	
k	surface roughness			✗						✗			✗
k	contraction coefficient							✗					
k	wave number										✗		
k_b	coefficient to be added to notch width							✗					
k_h	coefficient to be added to head							✗					
k_r	coefficient of particle friction											✗	
k_s	coefficient of total roughness to skin and form roughness											✗	
L	length	✗		✗	✗		✗	✗				✗	✗
L	wavelength										✗		
L_0	deep water wavelength										✗		
M	mass	✗	✗										
m	hydraulic mean depth					✗							✗
m	exponent in Ackers and White equation											✗	

		Used in chapters											
		1	2	3	4	5	6	7	8	9	10	11	12
N	runner speed of turbine or pump				✗								
N	periods of record								✗				
N	number of occurrences of event									✗			
N	number of waves in a storm										✗		
N_s	specific speed of turbine or pump				✗								
n	Manning's roughness coefficient					✗	✗						
n	exponent in flume equation							✗					
n	number of piers in spillway							✗					
n	ratio of wave group to individual wave velocities										✗		
n	exponent in Ackers and White equation											✗	
P	(intensity of) pressure	✗	✗	✗	✗								✗
P	wetted perimeter					✗	✗						
P	peak of spillway surface							✗					
P	probability								✗		✗		
P	wave power										✗		
p	height of weir							✗					
PIMP	percentage impermeable area									✗			
PR	percentage runoff								✗	✗			
Q	volume rate of discharge		✗	✗	✗	✗	✗	✗	✗	✗		✗	✗
Q_b	bedload flow rate											✗	
Q_s	total sediment flow rate											✗	

		Used in chapters											
		1	2	3	4	5	6	7	8	9	10	11	12
Q_{MED}	median annual maximum flow								✗	✗			
Q_{P}	peak runoff									✗			
q	volume flow rate per unit width					✗	✗						
q_{b}	bedflow sediment transport rate											✗	
R	hydraulic radius					✗	✗					✗	
R	radius							✗					
R	wave runup										✗		
$R\mathrm{s}$	shaft radius							✗					
r	rank order								✗				
r_{f}	factor relating to friction, incident wave angle, etc.												
RD	rainfall depth									✗			
Re	Reynolds number			✗								✗	✗
REST	residual soils term									✗			
S	hydraulic gradient			✗									
S_{F}	friction gradient						✗						
S_i	proportion of soil class within a catchment									✗			
S_0	bed slope					✗	✗					✗	
s	density ratio											✗	
SAAR	standard average annual rainfall								✗	✗			
SOIL	index of water holding capacity									✗			
SPR	standard percentage runoff								✗	✗			

		Used in chapters											
		1	2	3	4	5	6	7	8	9	10	11	12
T	return period								✗		✗		
T	wave period										✗		
T_r	round trip time			✗									
t	time		✗							✗	✗	✗	
t_C	time of concentration									✗			
t_E	time of entry									✗			
t_F	time of flow									✗			
T_P	time to peak								✗				
U_m	maximum orbital velocity of waves on bed											✗	
U_p	unit hydrograph peak flowrate								✗				
u_*	shear velocity											✗	
UCWI	urban catchment wetness index									✗			
URBAN	fraction of developed land within catchment									✗			
URBEXT	extent of urbanisation								✗				
V	velocity		✗	✗	✗	✗	✗	✗				✗	✗
V_c	critical velocity							✗				✗	
V_s	settling velocity											✗	
vol	volume	✗								✗			
vol_{XS}	excess runoff volume caused by development									✗			
W	weight	✗											
w	weight density	✗											
w	weighting factor								✗				
We	Weber number												✗

		Used in chapters											
		1	2	3	4	5	6	7	8	9	10	11	12
x	distance	✗	✗		✗			✗					
Y	plate separation			✗									
y	distance							✗					
y_s	scour depth											✗	
Z_1, Z_2	factors in Wallingford procedure									✗			
z	height above datum, elevation head		✗	✗			✗	✗					✗
α	velocity distribution coefficient					✗							
α	proportion of paved area draining to network or watercourse									✗			
α	angle of wave approach										✗		
α	notch angle							✗					
α	coefficient in Ackers and White equation											✗	
α	skew angle of approach											✗	
β	proportion of pervious area draining to network or watercourse									✗			
β	slope angle of beach										✗		
Γ	strength of a vortex												
ζ	surf similarity parameter										✗		
ζ_p	surf similarity factor based on peak period of wave spectrum										✗		
η	percentage of velocity maintained; efficiency				✗			✗					
θ	angle of inclination	✗											

		Used in chapters											
		1	2	3	4	5	6	7	8	9	10	11	12
θ	angle of jet							✗					
θ	tractive force coefficient (Shields parameter)											✗	
θ_c	critical value of tractive force coefficient											✗	
λ	friction factor			✗	✗								
μ	dynamic viscosity			✗									✗
ν	kinematic viscosity			✗								✗	
ρ	mass density	✗	✗	✗	✗	✗	✗				✗	✗	✗
σ	angular frequency of waves										✗		
τ	shear stress			✗								✗	✗
τ_c	critical shear stress					✗						✗	

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Chapter 1
Pressure in liquids: its effects and measurement

After studying this chapter you should be able to

- recall the relationships between density, mass and weight of a liquid and the units and symbols used for these
- calculate pressures at any depth in a liquid
- express pressures in terms of 'head' of different liquids
- describe common methods of measuring pressure within liquids
- calculate forces on horizontal, vertical and inclined plane immersed surfaces in a liquid and establish the depth to the centre of pressure
- calculate horizontal and vertical components of force on curved immersed surfaces and from these find the resultant force and its direction
- appreciate the relationship between centre of buoyancy, centre of gravity and metacentre and its role in establishing the stability of a floating body.

1.1. Introduction

Forces acting on bodies are of major interest to the engineer. This chapter shows how the value and location of forces acting on immersed bodies can be determined from a knowledge of the liquid properties and the geometry of the surfaces into which the liquid comes into contact. Methods of pressure measurement are introduced. Towards the end of the chapter, these techniques are used to determine the stability of floating bodies. While the emphasis within this chapter is on liquids at rest (hydrostatics), the methods of determining pressures and forces will be applicable to the consideration of liquids in motion (hydrodynamics) considered in the rest of the book.

1.2. Density and pressure within a liquid

1.2.1 Mass density

Mass density is denoted by ρ (the Greek letter rho). It is the mass of a substance divided by the volume (vol) it occupies:

$$\rho = \frac{M}{\text{vol}} \tag{1.1}$$

In SI units, M is measured in kg and vol in m^3, leading to ρ being in kg/m^3.

The mass density of freshwater is generally taken as $1000\,kg/m^3$, although there is slight variation with temperature. The maximum mass density of water occurs at 4°C. Salt water has a slightly higher mass density.

When the word 'density' is used without further qualification, it is usually taken to indicate 'mass density'.

1.2.2 Weight density

Weight is the force exerted on a mass in a gravitational field:

$$W = Mg \tag{1.2}$$

If M is in kg and g in m/s^2 W will be in newtons (N), a special name for kg · m/s^2.

In this book, g is taken as 9.81 m/s^2, although it needs to be recognised that this value is only approximate and varies from place to place. Because of this approximation, it is not sensible to quote answers to questions in which g is a factor to more than three significant figures.

The weight density w is given by

$$w = \rho g \tag{1.3}$$

This is also sometimes called the 'unit weight' or 'specific weight' of the liquid and its SI unit is N/m^3.

1.2.3 Hydraulic pressure

The intensity of pressure at a depth h in a liquid of density ρ is given by

$$P = \rho g h \tag{1.4}$$

In common use the words 'intensity of' are often omitted. It is important to appreciate that *hydraulic pressure is exerted equally in all directions*. We will usually express pressure relative to atmospheric pressure. Strictly speaking, we should refer to this relative pressure as gauge pressure, though this is often not done.

The SI unit of pressure is the pascal (Pa, a special name for N/m^2), although in civil engineering hydraulics applications, to avoid large numbers, it is usually convenient to convert final answers to kPa.

Example 1.1

Seawater in a tank 4 m deep has a density of 1025 kg/m^3. What pressure does this exert on the base of the tank?

$$P = \rho g h = 1025 \times 9.81 \times 4 = 40\,221 \text{ Pa} \tag{1.5}$$

Taking account of the approximate value of g used, this should be rounded to 40 200 Pa or 40.2 kPa.

1.2.4 Pressure head

Pressures are sometimes expressed in terms of 'head' h:

$$h = P/\rho g \tag{1.6}$$

This is the height of a column of a named liquid (often water or mercury) that would cause an equal pressure to that being measured.

Example 1.2

In Example 1.1 on seawater in a tank, what is the pressure head on the base of the tank

(*a*) in terms of metres of freshwater?
(*b*) in terms of metres of mercury, if the density of mercury is 13 600 kg/m^3?

$$(a) \qquad h = \frac{40\,221}{1000 \times 9.81} = 4.10\ \text{m head of freshwater} \tag{1.7}$$

$$(b) \qquad h = \frac{40\,221}{13\,600 \times 9.81} = 0.301\ \text{m head of mercury} \tag{1.8}$$

As can be seen from this example, because of mercury's high density, values expressed in metres head of mercury are usually very small. For this reason, they will often be quoted as millimetres head instead.

Key points

- The density of a liquid can be expressed as mass density or weight density. The word 'density' is often used by itself to indicate the former of these.
- The intensity of hydraulic pressure in a liquid is the unit weight of the liquid multiplied by the depth of liquid above the level at which the pressure is calculated.
- The words 'intensity of' are generally omitted.
- Hydraulic pressure acts equally in all directions.
- It is common to express pressure in terms of the height of a column of liquid, especially water or mercury, that would cause the same pressure as that being measured.

1.3. Measurement of pressure

1.3.1 Piezometers

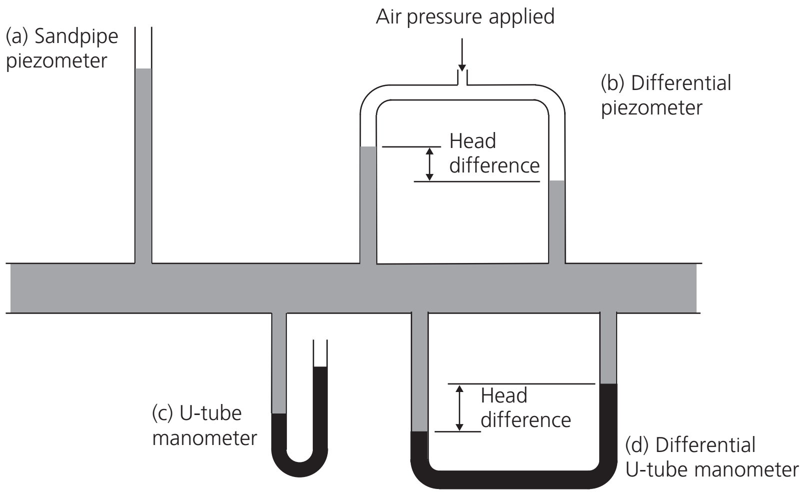
A piezometer is a simple means of measuring the pressure in a pipe. A small diameter tube is connected into the pipe and the height to which the liquid rises in the pipe gives the pressure head at the tapping point, as shown in Figure 1.1(a).

Sometimes it is the difference in pressure between two points that is of interest and two piezometers connected as illustrated in Figure 1.1(b) can be used, even if the actual pressure head extends beyond the height of the piezometer. This is achieved by applying air pressure to the top of the linking tubing. This will depress the liquid surface in the two tubes by an equal amount. The recorded head difference is, therefore, not affected by the application of the air pressure.

1.3.2 U-tube manometers

Pressure over the liquid in one limb of a manometer is balanced by a head of liquid in the other limb, as in Figure 1.1(c). Most commonly, the liquid in the second limb is mercury. The U-tube mercury manometer was invented by Poiseuille (1828). Because of mercury's high density, the height of tube needed is relatively small.

Figure 1.1 Piezometers and manometers



As long as the manometer liquid is designed appropriately so that the liquid is not forced round the bottom of the U, these manometers can be used in two circumstances in which the piezometer cannot: the measurement of pressures in gases and pressures below atmospheric pressure.

Pressure differences along a pipeline can be measured with a differential U-tube manometer, shown in Figure 1.1(d).

1.3.3 Mechanical pressure gauges

The most common of these is the Bourdon gauge. This consists essentially of a bent metal tube. When pressure is applied to the inside of the tube, it tends to straighten. The movement is transferred to an indicator needle on the face of the gauge.

1.3.4 Transducers

These work on the basis that a change in pressure stretches a wire; the resulting change in electrical resistance of the wire can be monitored electronically.

Key point

- Pressures can be measured using piezometers, mechanical gauges or transducers.

1.4. Forces on immersed surfaces

1.4.1 Introduction

The hydrostatic force F acting on the surface is the average intensity of pressure P multiplied by the surface area A over which it acts:

$$F = PA \tag{1.9}$$

1.4.2 Force acting on a horizontal immersed surface

Since all parts of the surface will be the same depth below the top of the liquid, the pressure will have a uniform value of ρgh across the surface and the force that acts perpendicularly to the surface will be obtained from the application of Equation 1.9.

Example 1.3

The tank holding the seawater in Examples 1.1 and 1.2 has a rectangular base 4 m wide and 5 m long. What is the force acting on the base?

$$F = 40\,221 \times 4 \times 5 = 804\,420\ \mathrm{N} \tag{1.10}$$

Again, the basic SI unit has a high numerical value and would probably be better expressed in kN. Hence, to three significant figures, the force acting on the base of the tank is 804 kN.

1.4.3 Force acting on a vertical immersed surface

For a surface of height h whose top is coincident with the top of the liquid, the pressure diagram will be triangular and the force will be given by

$$F = 0.5\rho ghA \tag{1.11}$$

For surfaces whose top is not coincident with the top of the liquid the pressure diagram will be trapezoidal and the force will be given by

$$F = \rho g\left(\frac{h_1 + h_2}{2}\right)A \tag{1.12}$$

where h_1 and h_2 are the depths of liquid above the top and base of the submerged surface.

Example 1.4

A 2 m wide rectangular screen is placed vertically within the tank of seawater previously considered. The top of the screen is situated 1 m below the water surface and the bottom of the screen is 3.5 m below the water surface. What is the hydrostatic force acting on one face of the screen?

$$\begin{aligned} F &= \rho g\left(\frac{h_1+h_2}{2}\right)A = 1025 \times 9.81 \times \left(\frac{1+3.5}{2}\right) \times (2 \times 2.5) = 113\,122\ \mathrm{N} \\ &= 113\ \mathrm{kN}\ (3\ \text{sig. figs}) \end{aligned} \tag{1.13}$$

For reasons that will become apparent later in this chapter, although it is a lengthier process, it is often useful to derive the force in the following way.

The pressure at the top of the screen:

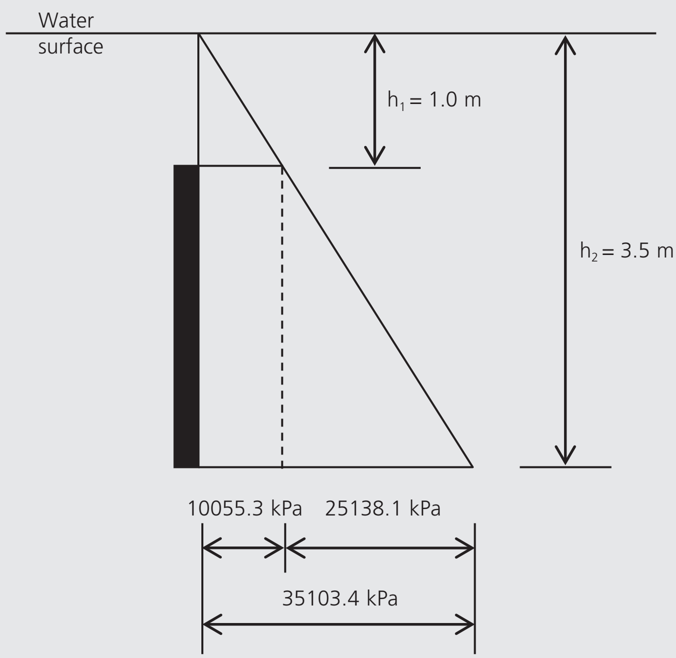
$$P_1 = 1025 \times 9.81 \times 1 = 10\,055.3\ \mathrm{Pa} \tag{1.14}$$

and the pressure at the bottom of the screen:

$$P_2 = 1025 \times 9.81 \times 3.5 = 35\,193.4 \text{ Pa} \tag{1.15}$$

These values are shown in Figure 1.2 as a sketch of pressure against depth. It can be seen that the pressure diagram can be split into rectangular and triangular sections.

Figure 1.2 Pressure distribution on submerged vertical surface



Applying

$$F = PA \tag{1.16}$$

to each section, the contributing elements of force are

$$\text{Rectangular section:}\quad F_R = 10\,055.3 \times (2 \times 2.5) = 50\,277 \text{ N} \tag{1.17}$$

$$\text{Triangular section:}\quad F_T = 0.5 \times 25\,138.1 \times (2 \times 2.5) = 62\,845 \text{ N} \tag{1.18}$$

The sum of these is 113 122 N, the same as previously found.

For a rectangular immersed surface, $(h_1 + h_2)/2$ is the depth h_G to the centroid, so the expression for force can be rewritten as

$$F = \rho g h_G A \tag{1.19}$$

In this form, the equation may be applied to surfaces of any shape, provided that h_G is known.

1.4.4 Location of resultant force acting on a vertical immersed rectangular surface

The point through which the resultant hydrostatic force F on an immersed surface acts is called the *centre of pressure*. As the pressure increases with depth, the centre of pressure will always be below the centroid of the immersed surface.

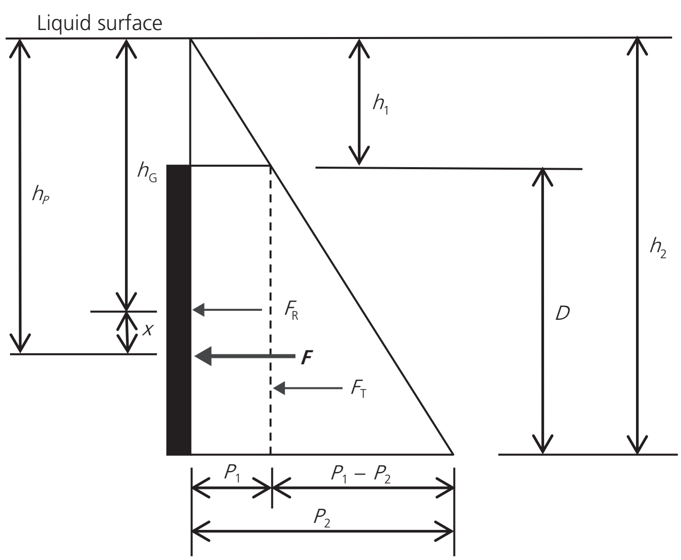
It was determined earlier in this chapter that the force may be considered as made up of two elements, F_R and F_T. F_R can be taken to act at the centre of the submerged surface and F_T at two-thirds of its height below its top.

If the centre of pressure lies at depth h_P below the water surface, we can take moments to find the distance x between the centroid and the centre of pressure (Figure 1.3).

If the submerged surface is B wide and D high and the pressures at the top and bottom of the surface are P_1 and P_2, respectively, then

$$F_R = P_1 BD = \rho g h_1 BD = \rho g \left(h_G - \frac{D}{2} \right) BD \tag{1.20}$$

Figure 1.3 Definition sketch for taking moments about surface of the liquid



and

$$F_{\mathrm{T}} = \frac{(P_2 - P_1)}{2} BD = \frac{\rho g(h_2 - h_1)}{2} BD = \frac{\rho g BD^2}{2} \tag{1.21}$$

The sum of the moments of F_{R} and F_{T} about the surface of the liquid equals the moment of the overall force:

$$F_{\mathrm{R}} h_{\mathrm{G}} + F_{\mathrm{T}}\left(h_{\mathrm{G}} + \frac{D}{6}\right) = (F_{\mathrm{R}} + F_{\mathrm{T}})(h_{\mathrm{G}} + x) \tag{1.22}$$

$$\therefore \quad \rho g\left(h_{\mathrm{G}} - \frac{D}{2}\right) BD \times h_{\mathrm{G}} + \frac{\rho g BD^2}{2} \times \left(h_{\mathrm{G}} + \frac{D}{6}\right) = \left(\rho g\left(h_{\mathrm{G}} - \frac{D}{2}\right) BD + \frac{\rho g BD^2}{2}\right) \times (h_{\mathrm{G}} + x) \tag{1.23}$$

This simplifies to

$$\left(h_{\mathrm{G}} - \frac{D}{2}\right) \times h_{\mathrm{G}} + \frac{D}{2} \times \left(h_{\mathrm{G}} + \frac{D}{6}\right) = \left(h_{\mathrm{G}} - \frac{D}{2} + \frac{D}{2}\right) \times (h_{\mathrm{G}} + x) \tag{1.24}$$

leading to

$$h_{\mathrm{G}}^2 - \frac{Dh_{\mathrm{G}}}{2} + \frac{Dh_{\mathrm{G}}}{2} + \frac{D^2}{12} = h_{\mathrm{G}}^2 + h_{\mathrm{G}} x \tag{1.25}$$

from which

$$x = \frac{D^2}{12 h_G} \tag{1.26}$$

Note that this distance between the centroid of the submerged surface and the centre of pressure is independent of the density of the liquid.

1.4.5 Location of resultant force acting on a vertical immersed surface of any shape

In general the width of the shape, as well as the pressure, varies with depth. For the general case of vertical surfaces, the depth h_P to the centre of pressure can be found using

$$h_P = h_{\mathrm{G}} + \frac{I_{\mathrm{G}}}{A h_{\mathrm{G}}} \tag{1.27}$$

where I_{G} is the second moment of area of the surface about an axis through its centroid. Most commonly encountered are rectangular and circular areas, for which values of I_{G} are shown in Table 1.1. Derivations of these are contained in most engineering mathematics textbooks, such as Stroud and Booth (2020).

Table 1.1 Second moment of area of rectangular and circular surfaces about an axis through their centroid

Shape	I_G
Rectangle of width B and height D	$BD^3/12$
Circle of radius R	$\pi R^4/4$