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MATHEMATICS IN TRANSPORT PLANNING AND CONTROL



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PREFACE

This volume contains forty of the papers presented at the International Conference on Mathematics in Transport Planning and Control which was held at Cardiff University from 1-3 April, 1998. This was the third such conference run under the auspices of the Institute of Mathematics and Its Applications, the first being held at the University of Surrey in 1979, and the second at Cardiff University in 1989. Over fifty papers were submitted for presentation at the conference, and the papers included in these Proceedings are those selected following a strict refereeing procedure.

It will be clear from the contents that mathematical ideas and methodologies continue to play a prominent part in the description and solution of the many and varied problems that are being currently investigated in quite diverse areas of transport research. There are papers relating to modes of transport such as road, rail, air and shipping. Applications will be found on transport planning, congestion, assignment, networks, signalling, road safety, and environmental issues. It is hoped that the reader, whether an "old hand" or a newcomer to the subject area, will find much to interest him/her. For those with little previous knowledge of the subject area, the Plenary papers presented by Professor R.E. Allsop (University College London) and Dr. R. Kimber (Transport Research Laboratory) are recommended as starting points.

I wish to express my personal thanks to the members of the organizing committee (Professor M. Maher, Dr. B.G. Heydecker, Dr. N. Hounsell, Dr. J.G. Hunt, Professor C. Wright), who also acted as Associate Editors, and to the referees who gave freely of their time and expertise. Finally, my thanks are due to my secretary, Ms June Thomas, without whose sterling efforts these Proceedings would not have seen the light of day.

J.D. Griffiths
Editor

ANALYSIS OF TRAFFIC CONDITIONS AT CONGESTED ROUNDABOUTS

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ABSTRACT

At roundabouts where entering traffic is required to give way to traffic circulating in the roundabout, the traffic capacity of each entry is a function of the flow of traffic circulating past it. This relationship has previously been analysed in two main ways: using a linear relationship based on regression and using a non-linear relationship based on a model of entering drivers' acceptance of gaps in the circulating traffic. The linear analysis has previously been extended to estimation of the reserve capacity or degree of overload of the roundabout as a whole in relation to a given pattern of approaching traffic. The non-linear analysis is extended similarly in this paper.

The relationships between entry capacity and circulating flow imply in turn that the capacity of each entry is a function of the entering flows and the proportions of traffic making various movements through the junction from some or all of the entries. Equations are established for determining derivatives of capacity or delay on each entry with respect to the demand flow for each movement. In particular, it is shown that when the roundabout is overloaded the capacity of an entry can depend upon the demand flow on that same entry, giving rise to a corresponding term in the derivative of the delay-flow relationship for the approach concerned.

1. GENERAL PRINCIPLES

Fundamental to the effectiveness of the roundabout as a road junction in resolving conflicts between the various traffic movements is the *offside priority rule*, whereby drivers of vehicles wishing to enter the roundabout are required to wait for gaps in, or *give way* to traffic already circulating in the roundabout. Having given way at this point, drivers then have priority for the rest of their passage through the junction. All merging and crossing conflicts between vehicular movements are thus resolved by requiring each driver to be ready to give way at one point, and only diverging conflicts remain for drivers to handle as they leave the circulating traffic at their chosen exits. Conflicts with pedestrian movements usually occur just before vehicles enter the junction or just after they leave it.

Vehicular traffic approaching a roundabout can be divided into identifiable sets called *streams* such that queuing theory can be applied to the traffic in each stream. With the offside priority rule, the *capacity* Q of a stream can be expected to be a decreasing function $F(q_c)$ of the *circulating flow*

2 Mathematics in Transport Planning and Control

q_c of traffic to which it should give way. The parameters of this function can depend upon the characteristics of the circulating traffic, and certainly depend upon the geometry of the junction, including the detailed layout of the entry concerned, but the latter is assumed to be such that the circulating flow for a stream does not include any traffic from other streams using the same entry. The value of q_c will thus depend on the approaching flows q_a and capacities in the streams at the preceding entries, and the proportions of traffic in these streams making each relevant movement through the junction. For each stream it is assumed that the *entering flow* is the lesser of the approaching flow and the capacity, and that the proportions of the entering vehicles making each movement are the same as those of the approaching vehicles. Flows and capacities are measured in passenger car units (pcu)/unit time, so that their values are approximately independent of the composition of the traffic.

The capacity of each stream is thus determined by the interaction between the whole pattern of traffic and the geometry of the junction, and for a given pattern of traffic the capacities of the various streams and the corresponding amounts of delay to traffic in them can be influenced by adjusting the geometrical layout.

Compliance with the offside priority rule should maintain free movement of circulating traffic so long as the exits from the roundabout do not become blocked. It is not, however, a guarantee of acceptable distribution of capacity and delay among the traffic streams, and can in extreme cases lead to disproportionate queues in certain streams as a result of lack of gaps in the corresponding circulating traffic. In such cases, it may be helpful to introduce signal control.

There are two main ways in which capacity can be analysed for a given geometrical layout.

- (a) For any given pattern of traffic, to estimate the resulting value of Q for each stream, thus enabling the queue-length and delay for the given approaching flow to be estimated for each stream by means of applied queueing theory. The results enable the given layout to be evaluated in terms of provision for the given pattern of traffic, and provide indications where attempts should be made to adjust the layout to provide more appropriate levels of capacity in different streams. They also enable the derivatives of capacity and delay in any one stream with respect to the amount of traffic wishing to make any particular movement through the junction from any stream to be estimated.

For this purpose, it is necessary to calculate the circulating flows past the various entries, each of which depends on the entering flows at preceding entries. Calculation of circulating flows is straightforward when the approaching flow in every stream is within capacity, because each entering flow is then equal to the corresponding approaching flow and is therefore known. When one or more streams is overloaded, however, the corresponding entering flows are equal to the as yet undetermined capacities of the corresponding streams, and a stepwise calculation is required.

- (b) For a given pattern of traffic, to calculate at what common multiple T^* , say, of the given approaching flows the most heavily loaded stream will have an approaching flow that is as close as is desirable in practice to its capacity. This is relevant to the question for how long, in a scenario of traffic growth, the junction will continue to function satisfactorily, and for which stream or streams difficulty will first arise.

For this purpose, suppose there are m vehicular traffic streams, and let the given approaching flow in stream i be q_{ai} ($i = 1, 2, \dots, m$). Then using the indicator of junction capacity first introduced by the author (Allsop, 1972) in the context of traffic signal control, and applied to roundabouts by Wong (1996), the junction is said to be working *at practical capacity for approaching flows proportional to the q_{ai}* when the arrival rates are T^*q_{ai} , and the *practical capacity of the junction for approaching flows proportional to the q_{ai}* is T^* times these approaching flows.

2. SPECIFICATION OF THE PATTERN OF TRAFFIC

For each stream i , let q_{ci} be the circulating flow to which traffic in stream i should give way. The flows q_{ai} and q_{ci} are flows that arise in the operation of the roundabout from the rates at which drivers wishing to make the various possible movements through the junction approach it in the various streams. To specify the demand for movement of vehicles through the junction in these terms, let a *movement* comprise traffic wishing to enter the junction in a particular stream and leave by a particular exit. Suppose that there are n distinct movements, and let the approaching flow in movement j be q_j ($j = 1, 2, \dots, n$).

Then following Wong (1996), but with a different choice of symbols, let \mathbf{a} be the $m \times n$ matrix (a_{ij}) such that

$$a_{ij} = \begin{cases} 1 & \text{if movement } j \text{ contributes to } q_{ai} \\ 0 & \text{if not} \end{cases}$$

and let \mathbf{c} be the $m \times n$ matrix (c_{ij}) such that

$$c_{ij} = \begin{cases} 1 & \text{if movement } j \text{ contributes to } q_{ci} \\ 0 & \text{if not} \end{cases}$$

Thus \mathbf{a} and \mathbf{c} specify which movements contribute to the approaching flow and circulating flow for each stream, and by the definition of a movement, \mathbf{a} has just one non-zero element in each column. It also follows that

$$q_{ai} = \sum_j a_{ij} q_j \tag{2.1}$$